Asset Pricing Implications of Social Networks*

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Abstract

Recent empirical studies suggest that social networks, according to which communication takes place, have a significant impact on traders' financial decisions. Motivated by this evidence, we propose an asset pricing model in which agents communicate information according to a social network. In the proposed model, agents initially have imperfect and diverse information about the asset payoff structure. Via communication, agents learn, i.e. gain information, about the asset payoff structure from others in the economy. The social network indicates whom each agent learns from. The social network is exogenous and can be considered to represent geographical proximities as well as social relationships (such as friendships and acquaintanceships). The model generates several novel implications. First, we prove that *social influence* is a determinant in asset pricing, where one's influence is determined by her connections in the social network. Then we show that proximities between agents in the social network affect agents' asset demand correlations: demands of agents from the same *tight-knit* social cluster exhibit higher correlations compared to demands of those from disjoint social clusters. Impact of social networks on asset price volatility is also explored. We demonstrate that learning in social networks may account for the observed high volatility ratio of price to fundamentals in the stock markets. Finally, we investigate how different specifications of social networks affect agents' assessments of the asset payoff structure. To that end, we introduce the notions of *informational dominance* and *informational efficiency*, which essentially rank social networks according to the precision of information they generate for agents in the economy. We provide partial characterizations of informationally dominant and informationally efficient social networks.

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1 Introduction

Communication of information among market participants plays an important role in financial decision-making. However, it is not just communication that affects financial decisions. The social network, according to which communication takes place, is also relevant. In this paper, we investigate the implications of social networks for asset prices and asset demands.

Our theoretical study of the subject is motivated by two strands of empirical studies. One strand focuses on the effects of communication in financial decision-making. Shiller and Pound (1989), Hong, Kubik and Stein (2004) can be counted among the studies of this strand. The first study suggests that communication with peers affects institutional investors' portfolio decisions. The latter shows that stock market participation is influenced by social interactions. The other strand of empirical studies, which motivates our paper, is concerned with the social network effects in financial decision-making. This strand includes Duflo and Saez (2002), Kelly and O'Grada (2000), Hong, Kubik, and Stein (2005). Duflo and Saez (2002) explore peer effects in retirement savings decisions. They find significant own-group peer effects on participation and on vendor's choice, but no cross-group peer effects.[1] Kelly and O'Grada (2000) investigate the behavior of Irish depositors in a New York bank during the panics of 1854 and 1857. The social networks of depositors are determined by place of origin in Ireland and neighborhood in New York. The paper shows that social network is the main factor in the determination of depositors' behaviors. Another study by Hong, Kubik and Stein (2005) exhibits that portfolio decisions of mutual fund managers are highly correlated if they are located in the same city and the decisions are lowly correlated if the managers are from different cities.

Motivated by this evidence, this paper proposes an asset pricing model which allows communication and learning to take place according to a social network. We consider a financial market economy, where two assets are traded: one risk-free and one risky. Agents initially have imperfect and diverse information about the risky asset payoff. Via communication,

[1] Also, Duflo and Saez (2003) analyze an experiment to study the role of information and social interactions in employees' retirement plan decisions. The results of the experiment are broadly consistent with the empirical findings in their earlier study, mentioned above.

agents learn, i.e., gain information, about the risky asset payoff from others in the economy. The social network indicates whom each agent learns from. The social network is exogenous and can be considered to represent geographical proximities as well as social relationships (such as friendships and acquaintanceships). In our proposed model, both risky asset price and social learning convey information across agents.

Given this setting, we study the existence of (linear) equilibrium and show that the equilibrium exists for any specification of social networks provided liquidity (net supply) of risky asset is sufficiently volatile. Then we study the asset pricing implications of social networks in the presence of highly volatile risky asset liquidity. First, we establish that *social influence* is a determinant in asset pricing: in particular, we show that the information of a socially influential agent, i.e., the information of an agent whom many learn from, has higher impact on the risky asset price compared to information of those with less influence. Second, we investigate how proximities between agents in the social network affect agents' asset demand correlations: we prove that demands of agents from the same *tight-knit* social cluster exhibit higher correlations compared to demands of those from disjoint social clusters. The notion of tight-knittedness is introduced to capture the fact that people mostly learn from those in their proximity and rarely from others. Note that, in this regard, our result is broadly consistent with the empirical study of Hong, Kubik and Stein (2005), mentioned above. The relation between risky asset price volatility and social networks is also explored. In particular, we discuss whether learning in social networks can account for the observed high volatility ratio of price to fundamentals. Our results on this issue are encouraging and indicate that social networks may indeed provide a justification for excess price volatility. Finally, we investigate how different specifications of social networks affect agents' assessments of the risky asset payoff. To that end, we introduce the notions of *informational dominance* and *informational efficiency*, which essentially rank social networks according to the precision of information they generate for agents in the economy. We provide partial characterizations of informationally dominant and informationally efficient social networks.

This paper is related to, and benefits from, several theoretical literatures, namely, the sociological literature on social networks, and the economic literatures on rational expectations and social learning. Our representation of social networks is based on the sociological literature

(French (1954), Harary (1959), Wasserman and Faust (1994)).[2] Most terms used throughout the text, such as *social influence*, *tight-knittedness*, and *centrality*, are borrowed from this literature. The asset pricing model proposed in this paper is based on the rational expectations literature (Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980)). Except the modelling of social networks, our description of the financial market economy mainly follows from Hellwig (1980). Also, the notion of learning used in this paper is closely related to the literature of social learning (Banerjee (1992), Bikchandani, Hirshleifer and Welch (1992), Ellison and Fudenberg (1993)). In the models of social learning theory, learning generally takes place through sequential observations of others' actions over time. Unlike these models, here we let agents learn from each other according to a social network in a static setup. Also, in this paper, learning takes place through observations of other agents' signals rather than sequential observations of their actions.

In relation to the problem studied and the model employed in this paper, three recent studies are especially worth mentioning: DeMarzo, Vayanos and Zwiebel (2003), Bisin, Horst and Ozgur (2005), and Ozsoylev (2004).

DeMarzo, Vayanos and Zwiebel (2003) propose a boundedly-rational model of opinion formation in social networks. The pivotal assumption of their model is *persuasion bias*, which refers to agents' failure to adjust for possible repetitions of (or common sources in) information they receive. This assumption leads to *social influence*: agents, who are "well-connected" in the social network, may have more influence in the overall formation of opinions in the economy regardless of their information accuracies. The scope of DeMarzo, Vayanos and Zwiebel (2003) is much broader than this paper, however we still obtain a result similar to theirs regarding social influence: we show that the information of an agent, whom many learn from via communication, has higher impact on the asset price compared to the information of agents with fewer connections in the social network. Unlike DeMarzo, Vayanos and Zwiebel (2003), we do not dispense with rationality.

Another closely related paper is Bisin, Horst and Ozgur (2005): they provide a model in which rational agents interact locally. The utilities of agents are modelled so that they depend

[2] French (1954) and Harary (1959) are seminal sociological papers studying social power and its relation to the structure of social network. Wasserman and Faust (1994) is a classic text in this field.

on the actions of those that they interact with. The paper analyzes only one-sided interactions (i.e., bilateral interactions among agents are not included in the analysis) and assumes that each agent interacts with exactly one other agent. Local preferences for conformity and habit persistence are established in the paper. The scope of Bisin, Horst and Ozgur (2005) and the scope of our paper do not completely overlap: in particular, Bisin, Horst and Ozgur (2005) do not study financial markets where asset prices convey information on top of social network. Also, our analysis is not confined to one-sided interactions or interactions with exactly one agent. Actually, in our paper there are no restrictions on the social network structure (hence no restrictions on the nature of social interactions). Having said that, we should note the broader specification of utility functions in Bisin, Horst and Ozgur (2005), where CARA utility (the specification of utility function in our paper) is just a special case. Also, unlike ours, they have a dynamic setup.

Ozsoylev (2004) is closest to this paper in regards to the problem studied and the model employed: the paper proposes a financial market model where agents learn from other's actions (asset demands). Whom an agent learns from is determined by a social network. Both asset prices and social learning convey information. Most of the analysis in the paper is confined to one-sided interactions, but unlike the setting of Bisin, Horst and Ozgur (2005) agents do not need to learn from exactly one agent. For instance, the setup allows for some agents to learn from no other agents and actions of some to be observed by multiple agents. Several asset pricing implications are obtained when social networks impose hierarchy in interactions (i.e., when networks are in the form of trees). These implications are similar to the ones we obtain here: social influence is proved to be a determinant in asset pricing when the social network imposes hierarchy, and proximities in *highly stylized* social networks are shown to affect asset demand correlations. The main difference between Ozsoylev (2004) and this paper is in the nature of learning: here learning takes place through observations of others' signals rather than their asset demands. Hence, the analysis of equilibrium in this paper is significantly simpler compared to Ozsoylev (2004) because signals are exogenous to the model whereas asset demands are endogenous. This simplification allows us to extend the equilibrium analysis to all possible specifications of social networks, and obtain several novel asset pricing implications. Also, this paper complements Ozsoylev (2004) by investigating asset pricing under a different form of social learning.

This paper is organized as follows. Section 2 exhibits our asset pricing model which allows communication and learning to take place according to a social network. This section also provides the definition of equilibrium. The results of equilibrium analysis is given in Section 3. Section 3.1 studies the existence of equilibrium. Section 3.2 analyzes the relation between social influence and asset prices. Section 3.3 shows that proximities in the social network affect asset demand correlations. Section 3.4 explores whether learning in social networks can account for excess price volatility in the stock markets. Section 3.5 investigates the effect of social networks on informational efficiency.

2 The Basic Model

In this section, we model the economy and define its equilibrium. Except the modelling of communication and learning in social networks, the description of the economy is mainly based on Hellwig (1980).

The economy lasts for two periods. There are $n \geq 2$ agents, indexed by $i = 1, ..., n$. Trade takes place in the first period and consumption of a single good in the second. A risk-free asset and a risky asset are traded. The risky asset has a future random[3] payoff $\tilde{X}$, which realizes in the second period. The price and the payoff of the risk-free asset are normalized to 1. We let p be the price of the risky asset.[4] Each agent i is endowed with deterministic wealth w_{0i} (in units of consumption good). If agent i purchases z_i units of the risky asset, her portfolio yields the random final wealth $\tilde{W}_i = z_i\tilde{X} + (w_{0i} - pz_i)$.

For $i = 1, ..., n$, agent i has a CARA utility function $u(c) = -\exp(-\rho\, c)$. The parameter $\rho \in (0, \infty)$ denotes the absolute risk aversion coefficient, which is common for all agents. Agent i's expected utility of final wealth is given by $\mathrm{E}_i[u(\tilde{W}_i)] = \mathrm{E}_i[-exp(-\rho\, \tilde{W}_i)]$. The expectation operator, E_i, is conditional on agent i's information $\mathcal{I}_i$.

[3]Throughout the text, we use the following convention: random variables are denoted with *tilde* (such as $\tilde{y}$), and the realizations of random variables are denoted without *tilde* (such as y).

[4]Here, and throughout the paper, the terms *price* and *demand* are exclusively used for the risky asset price and demand, respectively, unless otherwise stated.

Each agent i observes the realization of a private random signal $\tilde{\theta}_i = \tilde{X} + \tilde{\epsilon}_i$. Signal $\tilde{\theta}_i$ communicates the risky payoff $\tilde{X}$ perturbed by some noise $\tilde{\epsilon}_i$.

The liquidity (i.e., the net supply) of the risky asset is L, which is taken to be the realization of a random variable $\tilde{L}$. We assume that the random vector $(\tilde{X}, \tilde{L}, \tilde{\epsilon}_1, ..., \tilde{\epsilon}_n)$ is normally distributed with mean $(\mu_x, 0, 0, ..., 0)$, and nonsingular variance-covariance matrix $({\sigma_x}^2, {\sigma_L}^2, {\sigma_\epsilon}^2, ..., {\sigma_\epsilon}^2)\, \imath_{n+2}$, where $\imath_{n+2}$ denotes the $(n+2)$ dimensional identity matrix. Note that the last assumption implies homogeneity in signal precisions.

In our model, agents learn, i.e., gain information, from other agents about the risky payoff $\tilde{X}$ via communication. Whom an agent learns from is determined by a social network. The social network is modelled by a simple directed graph,[5] with vertices representing the agents, and directed edges representing the directions of learning. Agents learn from those to whom they are linked by a directed edge in the social network: $i \to j$ denotes that i learns from j about the risky payoff $\tilde{X}$.

We have described from whom agents learn in the economy. Next we describe what these agents learn. To that end, we need to introduce a couple of notations. Let Ω be the set of all possible simple directed graphs with n many vertices. For any given social network $\mathcal{N} \in \Omega$ and agent $i \in \{1, ..., n\}$, we let $\mathcal{S}(\mathcal{N}, i)$ denote the following set

$$\{k \in \{1, ..., n\} : i \to k\} \cup \{i\}.$$

Note that the vector $(\mathcal{S}(\mathcal{N}, 1), \mathcal{S}(\mathcal{N}, 2), ..., \mathcal{S}(\mathcal{N}, n))$ fully captures the interactions among agents imposed by the social network $\mathcal{N}$. We refer to $k \in \mathcal{S}(\mathcal{N}, i)$ as an *information source* for agent i. By definition, an agent is always an information source for herself. Our model assumes that learning takes place for any given agent i through the observation of the following parameter:

$$\mathrm{E}\left[\tilde{X} \middle| \{\theta_k\}_{k \in \mathcal{S}(\mathcal{N}, i)}\right].$$

That is, each agent i observes the expectation of risky payoff $\tilde{X}$ conditional on her information sources' private signals. We refer to $\mathrm{E}\left[\tilde{X} \middle| \{\theta_k\}_{k \in \mathcal{S}(\mathcal{N}, i)}\right]$ as agent i's *social inference*. Note

[5] A graph is called *simple* if multiple edges between the same pair of vertices or edges connecting a vertex to itself are forbidden. A graph is called *directed* if edges exhibit inherent direction, implying every relationship so represented is asymmetric.

that when $\mathcal{S}(\mathcal{N},i)=\{i\}$, social inference $\mathrm{E}[\tilde{X}\big|\theta_i]$ does not communicate any information on top of private signal θ_i. This is sensible, because $\mathcal{S}(\mathcal{N},i)=\{i\}$ means that agent i does not learn from others in the social network.

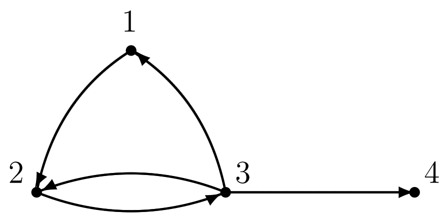

Figure 1: ILLUSTRATION OF A SOCIAL NETWORK

This social network, $\mathcal{N}$, assigns the following information sources to agents $i=1,...,4$: $\mathcal{S}(\mathcal{N},1)=\{1,2\}$, $\mathcal{S}(\mathcal{N},2)=\{2,3\}$, $\mathcal{S}(\mathcal{N},3)=\{1,2,3,4\}$, $\mathcal{S}(\mathcal{N},4)=\{4\}$. According to this network, agent 3 learns about the risky payoff from all agents in the economy while agent 4 learns from none.

Is our modelling of social learning plausible? In particular, why should we regard the parameter $\mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big]$ as a good representation of agent i's actual inference from a social learning process? Because $\mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big]$ aggregates all information from agent i's sources that is relevant to $\tilde{X}$. Also, this information is aggregated efficiently in the following sense: $\mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big]$ is a sufficient statistic for agent i's information sources' signals. Formally speaking, the conditional distribution of risky payoff $\tilde{X}$ given $\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}$ is the same as the conditional distribution of $\tilde{X}$ given $\mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big]$.[6]

In our economy, the information $\mathcal{I}_i$, on which agent i conditions her expectation of $\tilde{X}$, consists of the realized private signal θ_i, social inference $\mathrm{E}\left[\tilde{X}\middle|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right]$ and price p. So both social inference and price convey information across agents in the economy.

Price p that prevails in the market depends on the realized liquidity L and the private signals $\theta_1,...,\theta_n$. Considering the whole range of realizations of the random variables $\tilde{L}$ and $\tilde{\theta}_1,...,\tilde{\theta}_n$, the realized market prices generate a random variable $\tilde{p}$. We assume the following:

[6]To see this, check equation 4.9 in the appendix. It reveals that $\mathrm{E}\left[\tilde{X}\middle|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right]$ is a linear function of $\sum_{k\in\mathcal{S}(\mathcal{N},i)}\tilde{\theta}_k$. It is well-known that any linear function of $\sum_{k\in\mathcal{S}(\mathcal{N},i)}\tilde{\theta}_k$ is a sufficient statistic for signals $\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}$ when signals are jointly normal and iid. That is, the conditional distribution of risky payoff $\tilde{X}$ given $\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}$ is the same as the conditional distribution of $\tilde{X}$ given any linear function of $\sum_{k\in\mathcal{S}(\mathcal{N},i)}\tilde{\theta}_k$.

for $i=1,...,n$, agent i knows the joint distribution of the random vector

$$\left(\tilde{X}, \tilde{\theta}_i, \mathrm{E}\left[\tilde{X}\middle|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], \tilde{p}\right).$$

This assumption imposes the hypothesis that expectations, determined through the private signals, social inferences and price, are rational.

Finally, we assume that all agents, $i=1,...,n$, are price takers.

For the described economy, we define the equilibrium in the fashion of rational expectations: *an equilibrium consists of a risky asset price function $P(\theta_1,...,\theta_n,L)$ and demand functions $\left\{z_i\left(\theta_i, \mathrm{E}\left[\tilde{X}\middle|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], p\right)\right\}_{i=1,...,n}$ such that for all realizations $(\theta_1,...,\theta_n,L)$ of $(\tilde{\theta}_1,...,\tilde{\theta}_n,\tilde{L})$*

(a) agent i's demand $z_i\left(\theta_i, \mathrm{E}\left[\tilde{X}\middle|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], p\right)$ maximizes her expected utility of final wealth $\mathrm{E}\left[u_i(\tilde{w}_{1i})\middle|\theta_i, \mathrm{E}\left[\tilde{X}\middle|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], p=P(\theta_1,...,\theta_n,L)\right]$ for all $i=1,...,n$,

(b) market clears, i.e., $\sum_{i=1}^{n} z_i\left(\theta_i, \mathrm{E}\left[\tilde{X}\middle|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], p\right)=L$.

In our equilibrium analysis, we will be focusing on equilibrium prices that are linear in private signals and liquidity, that is, equilibrium prices of the form

$$P(\tilde{\theta}_1,...,\tilde{\theta}_n,\tilde{L})=\pi_0+\sum_{i=1}^{n}\pi_i\tilde{\theta}_i-\gamma\tilde{L}.$$

This practice, i.e., exclusive analysis of linear equilibria, is common in the rational expectations literature, because non-linearity significantly reduces tractability.

3 Equilibrium Analysis

3.1 Existence of Equilibrium

The first thing we investigate is the existence of equilibrium in the described economy. Our existence proof is by construction: construct a linear function relating the risky asset price to signals and liquidity and show that this function is indeed an equilibrium price function.

As we noted before, the precursor of our model is Hellwig (1980), which effectively assumes that agents are socially isolated, i.e., $\mathcal{S}(N,i)=\{i\}$ for all $i=1,...,n$. From the aspect of social inference, our model and equilibrium analysis naturally encompass those in Hellwig. Hellwig (1980) cannot obtain an equilibrium in closed form, and neither can we. Also, our existence proof needs to tackle an added complexity: in standard rational expectations models, including Hellwig's, only price conveys information across agents, whereas in our model both price and social inference convey information. These two conveyers provide different and sometimes even conflicting information to agents, which complicates the analysis of conditional expectation formation regarding the risky payoff. Large liquidity variance reduces the complexity brought by multiple information conveyers in the following sense. When the liquidity variance is large, variations in price reflect variations in liquidity rather than variations in signals. In this case, social inference becomes a more reliable predictor of risky payoff compared to price, hence agents form their expectations and determine their demands mostly based on social inference. Making use of this observation, we prove that a linear equilibrium price exists for any specification of social networks if the liquidity variance is sufficiently large. Formally, we have the following result:

Proposition 1 *If the liquidity variance σ_L^2 is sufficiently large, then for any given social network $\mathcal{N} \in \Omega$ there exists a linear equilibrium price of the form*

$$\tilde{p}^{\mathcal{N}} \;=\; P(\tilde{\theta}_1, ..., \tilde{\theta}_n, \tilde{L}) \;=\; \pi_0^{\mathcal{N}} + \sum_{i=1}^{n} \pi_i^{\mathcal{N}}\, \tilde{\theta}_i - \gamma^{\mathcal{N}}\, \tilde{L} \tag{3.1}$$

with price coefficients that satisfy

$$0 \;<\; \lim_{\sigma_L^2 \to \infty} \gamma^{\mathcal{N}} \;<\; \infty, \qquad 0 \;<\; \lim_{\sigma_L^2 \to \infty} \pi_i^{\mathcal{N}} \;<\; \infty, \quad i = 1, ..., n. \tag{3.2}$$

Proposition 1 shows the existence of a linear equilibrium price satisfying certain limit characteristics: in the limit as $\sigma_L^2 \to \infty$, the linear equilibrium price assigns strictly positive finite coefficients to agents' private signals and a strictly negative finite coefficient to liquidity. It is sensible that private signals are assigned positive coefficients because the higher the realized private signals the higher the risky asset demands and hence the risky asset price should be higher. The higher the liquidity the lower the price should be, which justifies the strictly negative coefficient for liquidity. Also, if one of the limit coefficients were infinite, then the risky asset price would be $\pm\infty$. Liquidity is independent from the risky payoff and higher liquidity yields lower risky asset price. Thus there is no reason to justify an infinite risky asset price in the limit as $\sigma_L^2 \to \infty$. In light of these observations, the limit characteristics mentioned above seem quite plausible.

In sections 3.2-3.5, we analyze implications of learning in social networks on the risky asset price and demands. This analysis assumes that the liquidity variance σ_L^2 is large and it only focuses on the equilibrium prices satisfying (3.1) and (3.2). Since such equilibria are shown to exist in the presence of sufficiently liquidity variance, the forthcoming results are not vacuous.

3.2 Price and Social Influence

How do asset prices aggregate information in the presence of learning in social networks? In particular, we are concerned with the relative impacts of private signals on the equilibrium price in each given social network $\mathcal{N}$, i.e., the coefficients $\pi_i^{\mathcal{N}}$, $\pi_{i'}^{\mathcal{N}}$ with which the signals $\theta_i, \theta_{i'}$ affect the equilibrium price. In the case of complete social isolation, i.e., when $\mathcal{S}(\mathcal{N}, i) = \{i\}$ for all $i = 1, ..., n$, Hellwig (1980) shows that all private signals have the same impact on price as long as agents are homogenous in signal precision and risk aversion. Recall that our model assumes homogeneity in these two factors. Therefore, in our analysis, if any disparity arises among the signal impacts on price, this has to be due to the heterogeneity brought by social networks. Such heterogeneity can come through two routes: agents may have different numbers of information sources, or agents may be information sources for different numbers of agents. The heterogeneity due to the latter case can be interpreted as the heterogeneity in "social influence". The following proposition shows that social influence affects asset pricing, in particular, the aggregation of information within asset prices.

Proposition 2 *For a given social network $\mathcal{N} \in \Omega$ let $\tilde{p}^{\mathcal{N}}$ be an equilibrium price satisfying (3.1) and (3.2). Consider a set of agents $\{i, i'\} \subset \{1, ..., n\}$ such that $\mathcal{S}(\mathcal{N}, i) \supsetneq \{i\}$ and $\mathcal{S}(\mathcal{N}, i') \supsetneq \{i'\}$. If*

$$\sum_{m=1}^{n} \left|\mathcal{S}(\mathcal{N}, m) \cap \{i\}\right| > \sum_{m=1}^{n} \left|\mathcal{S}(\mathcal{N}, m) \cap \{i'\}\right|,$$

then $\pi_i^{\mathcal{N}} > \pi_{i'}^{\mathcal{N}}$ for sufficiently large liquidity variance σ_L^2.

Proposition 2 says the following: if (i) the liquidity variance is sufficiently large, (ii) both agents, i and i', learn from others, (iii) compared to agent i', agent i is an information source for a higher number of agents, then agent i's private signal has a higher impact on price compared to signal of i' . The intuition for this result is as follows. As we elaborated before in §3.1, when the liquidity variance is large agents determine their demands mostly based on social inference. In this case, if agent i is an information source for a higher number of people compared to i', agent i's signal affects a higher number of agents' demands through social inference and hence i's signal should have higher impact on price.

In summary, the higher the agent's social influence, the higher will be her signal's impact on price.

3.3 Demand Correlations and Tight-Knit Social Clusters

An empirical study by Hong, Kubik, and Stein (2004a) shows that mutual-fund managers are heavily influenced by the decisions of other fund managers working in the same city: a fund manager is more likely to hold (or buy, or sell) a particular stock in any quarter if other managers from different fund families located in the same city are holding (or buying, or selling) that same stock. The authors interpret this using an epidemic model where investors spread information about stocks directly to one another by word of mouth.

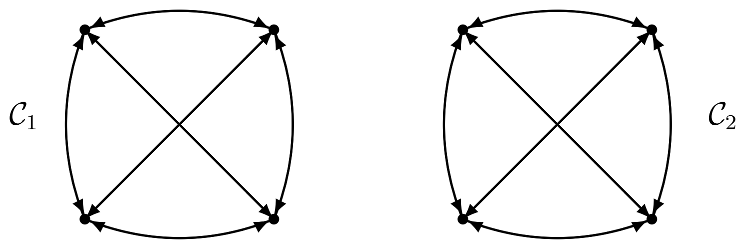

Figure 2: TIGHT-KNIT SOCIAL CLUSTERS

This empirical finding suggests that social networks play a relevant role in the determination of asset demands. In particular, these studies suggest that demands from the same "tight-knit" social cluster are highly correlated whereas demands from disjoint social clusters have insignificant correlation. In this paper, we categorize a social cluster as "tight-knit" if (i) everyone in the cluster learns from all of the agents in that cluster and no one else, and (ii) everyone in the cluster is an information source for each of the agents in that cluster and no one else. Formally, a social cluster $\mathcal{C} \subset \{1, ..., n\}$ is *tight-knit* if

$$\mathcal{S}(\mathcal{N}, i) = \{m : \mathcal{S}(\mathcal{N}, m) \cap \{i\} \neq \emptyset\} = \mathcal{C}, \quad \forall\, i \in \mathcal{C}.$$

The above interpretation of a tight-knit social cluster is somewhat stark, but it captures the crucial element of information transmission within the geographically proximate populations: people mostly learn from those in their proximity and rarely learn from others.

The next result partially reveals the relation between demand correlations and proximity in social networks. Our theoretical finding is broadly consistent with the empirical studies discussed above.

Proposition 3 *For a given social network $\mathcal{N} \in \Omega$ let $\tilde{p}^{\mathcal{N}}$ be an equilibrium price satisfying (3.1) and (3.2). For $i = 1, ..., n$, let $\tilde{z}_i^{\mathcal{N}} := z_i\left(\tilde{\theta}_i, \mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k \in \mathcal{S}(\mathcal{N}, i)}\big], \tilde{p}^{\mathcal{N}}\right)$ denote the equilibrium demand at $\tilde{p}$. Choose two tight-knit social clusters $\mathcal{C}_1 \subsetneq \{1, ..., n\}$ and $\mathcal{C}_2 \subsetneq \{1, ..., n\}$ such that $\mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset$ and $|\mathcal{C}_1| = |\mathcal{C}_2| > 1$. If the liquidity variance σ_L^2 is sufficiently large and $i \in \mathcal{C}_1$, then for any $h \in \mathcal{C}_1$ and $h' \in \mathcal{C}_2$*

$$\mathrm{corr}(\tilde{z}_i^{\mathcal{N}}, \tilde{z}_h^{\mathcal{N}}) \ > \ \mathrm{corr}(\tilde{z}_i^{\mathcal{N}}, \tilde{z}_{h'}^{\mathcal{N}}).$$

Proposition 3 compares demand correlations within and across the social clusters which are (i) tight-knit, (ii) non-degenerate[7], (iii) identical, and (iv) disjoint. The proposition says that the correlation of demands within the same social cluster is larger than the correlation of demands across different social clusters satisfying (i)-(iv) if the liquidity variance is sufficiently large. The intuition is straightforward. Due to large liquidity variance, agents determine their demands mostly based on social inference. Each agent's social inference is confined by the social cluster she is in. Since the cluster is tight-knit, the information conveyed by social inference across different agents of the cluster is highly correlated. This, in turn, implies

[7] i.e., the cluster includes more than one agent.

highly correlated demands within the cluster. On the other hand, the information carried by social inference to agents from disjoint clusters has little correlation. Thus, demands across disjoint clusters are lowly correlated.

3.4 Excess Price Volatility and Social Networks

Empirical studies, including Shiller (1981) and Mankiw, Romer and Shapiro (1985,1991), reveal that stock prices are more volatile compared to stock fundamentals. Different theories have been proposed to explain the discrepancy in the volatilities. Campbell and Kyle (1993) suggest that noise trading can be the factor creating discrepancy. This argument works in our setup as well: when we consider an equilibrium price that satisfies (3.1) and (3.2), we can easily see that the variance of the equilibrium price exceeds the variance of the risky payoff as long as the liquidity is sufficiently volatile. However, the high volatility ratio of price to fundamentals cannot be empirically justified only by high liquidity variance (or large noise trading). In this section we are investigating how social networks impact price volatility and whether networks can provide an alternative justification for the observed volatility ratio of price to fundamentals.

The analysis of linear equilibrium allows for a natural decomposition of the price volatility into information driven and liquidity driven components. In particular, given an equilibrium price of the form $\tilde{p}^{\mathcal{N}} = \pi_0^{\mathcal{N}} + \sum_{i=1}^{n} \pi_i^{\mathcal{N}} \, \tilde{\theta}_i - \gamma^{\mathcal{N}} \, \tilde{L}$, the price volatility is delivered by

$$\mathrm{var}(\tilde{p}^{\mathcal{N}}) \;=\; \left(\sum_{j=1}^{n} \pi_j^{\mathcal{N}} \right)^2 \sigma_x^2 + \sum_{j=1}^{n} (\pi_j^{\mathcal{N}})^2 \sigma_\epsilon^2 + (\gamma^{\mathcal{N}})^2 \sigma_L^2,$$

where the *information driven volatility component* is

$$\mathrm{var}_I(\tilde{p}^{\mathcal{N}}) \;:=\; \mathrm{var} \left(\sum_{i=1}^{n} \pi_i^{\mathcal{N}} \, \tilde{\theta}_i \right) \;=\; \left(\sum_{j=1}^{n} \pi_j^{\mathcal{N}} \right)^2 \sigma_x^2 + \sum_{j=1}^{n} (\pi_j^{\mathcal{N}})^2 \sigma_\epsilon^2,$$

and the *liquidity driven volatility component* is

$$\mathrm{var}_L(\tilde{p}^{\mathcal{N}}) \;:=\; \mathrm{var} \left(\gamma^{\mathcal{N}} \, \tilde{L} \right) \;=\; (\gamma^{\mathcal{N}})^2 \sigma_L^2.$$

The following proposition shows the dependence of price volatility and volatility components on the social network structure.

Proposition 4 *Let $\tilde{p}^{\mathcal{N}_1}$ and $\tilde{p}^{\mathcal{N}_2}$ be equilibrium prices for social networks $\mathcal{N}_1 \in \Omega$ and $\mathcal{N}_2 \in \Omega$, respectively, satisfying (3.1) and (3.2). Suppose that $\mathcal{S}(\mathcal{N}_1, i) \supsetneq \{i\}$ and $\mathcal{S}(\mathcal{N}_2, i) \supsetneq \{i\}$ for all $i = 1, ..., n$. For sufficiently large liquidity variance σ_L^2,*

- *(a)* $\mathrm{var}_L(\tilde{p}^{\mathcal{N}_1}) < \mathrm{var}_L(\tilde{p}^{\mathcal{N}_2})$ *if* $\sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_1, i)\right| > \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_2, i)\right|$,
- *(b)* $\mathrm{var}_I(\tilde{p}^{\mathcal{N}_1}) > \mathrm{var}_I(\tilde{p}^{\mathcal{N}_2})$ *if* $\sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_1, i)\right| = \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_2, i)\right|$ *and* $\sum_{i=1}^n \left(\sum_{m=1}^n \left|\mathcal{S}(\mathcal{N}_1, m) \cap \{i\}\right|\right)^2 > \sum_{i=1}^n \left(\sum_{m=1}^n \left|\mathcal{S}(\mathcal{N}_2, m) \cap \{i\}\right|\right)^2$,
- *(c)* $\mathrm{var}(\tilde{p}^{\mathcal{N}_1}) < \mathrm{var}(\tilde{p}^{\mathcal{N}_2})$ *if* $\sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_1, i)\right| > \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_2, i)\right|$.

Let us first discuss the interpretations of the conditions given in Proposition 4. The condition $\sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_1, i)\right| > \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_2, i)\right|$ simply says that, on average, agents in $\mathcal{N}_1$ have higher number of information sources compared to those in $\mathcal{N}_2$. The dual condition

$$\begin{aligned} \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_1, i)\right| &= \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_2, i)\right| \\ \sum_{i=1}^n \left(\sum_{m=1}^n \left|\mathcal{S}(\mathcal{N}_1, m) \cap \{i\}\right|\right)^2 &> \sum_{i=1}^n \left(\sum_{m=1}^n \left|\mathcal{S}(\mathcal{N}_2, m) \cap \{i\}\right|\right)^2 \end{aligned} \tag{3.3}$$

implies that social network $\mathcal{N}_1$ has more "central nodes" compared to $\mathcal{N}_2$. Rigorously speaking, this means the following: (i) agents on average have the same number of information sources in $\mathcal{N}_1$ as in $\mathcal{N}_2$, (ii) compared to $\mathcal{N}_2$ there are more agents in $\mathcal{N}_1$ who act as information sources to a number higher than the average number in the given network. We say that social network $\mathcal{N}_1$ is *more centralized* than social network $\mathcal{N}_2$ if the dual condition (3.3) holds.

Figure 3: CENTRALIZATION IN SOCIAL NETWORKS

Social network $\mathcal{N}_1$ is more centralized than social network $\mathcal{N}_2$. Agent 1 is the central node in $\mathcal{N}_1$.

Proposition 4 compares price volatility and volatility components across two social networks, namely $\mathcal{N}_1$ and $\mathcal{N}_2$, in which agents learn from at least one other agent. The proposition considers only the case where the liquidity variance is large, which implies that agents determine their demands mostly based on social inference. Under these restrictions, the following results are obtained:

(a) the liquidity driven volatility component is smaller in social network $\mathcal{N}_1$ than that in social network $\mathcal{N}_2$ if on average agents in $\mathcal{N}_1$ have higher number of information sources compared to those in $\mathcal{N}_2$,

(b) the information driven volatility component is greater in social network $\mathcal{N}_1$ than that in social network $\mathcal{N}_2$ if network $\mathcal{N}_1$ is more centralized than network $\mathcal{N}_2$,

(c) the price volatility is smaller in social network $\mathcal{N}_1$ than that in social network $\mathcal{N}_2$ if on average agents in $\mathcal{N}_1$ have higher number of information sources compared to those in$\mathcal{N}_2$.

The intuitions for these results are as follows. If agents have higher number of information sources in $\mathcal{N}_1$ compared to $\mathcal{N}_2$, then agents have more information available to them in $\mathcal{N}_1$ compared to $\mathcal{N}_2$ through social inference. More information helps agents better disentangle liquidity from price, which consequently decreases the impact of liquidity on price. Thus, the liquidity driven volatility component is smaller in $\mathcal{N}_1$ than that in $\mathcal{N}_2$. This explains part (a). The intuition for part (c) is straightforward in light of this explanation: since the proposition only focuses on an economy with large liquidity variance, the size of liquidity driven volatility component surpasses that of the information driven volatility component, which effectively means price volatility is determined mostly by the liquidity driven volatility component. Hence, we have the result given in (c). If social network $\mathcal{N}_1$ is more centralized than $\mathcal{N}_2$, then agents in $\mathcal{N}_1$ determine their demands mostly based on the central nodes' signals whereas agents in $\mathcal{N}_2$ treat all signals in their social inferences relatively equally while forming their demands. This means in network $\mathcal{N}_1$ changes in price are significantly dependent on changes in central nodes' signals. Whereas in network $\mathcal{N}_2$ changes in price are relatively equally dependent on changes in each signal, and given the independence between the error terms of the signals most of these changes wash out each other. Therefore, the information driven volatility component in $\mathcal{N}_1$ exceeds that of $\mathcal{N}_2$. This explains part (b).

Recall that the initial motivation of this section was to see whether learning in social networks can account for the observed high price-to-fundamental volatility ratio. Virtually all theoretical and empirical studies on price volatility to date assume that agents are socially isolated, i.e., that they do not learn from each other. In light of this, Proposition 4 seems to enhance the puzzle regarding excess price volatility because the proposition implies that price volatility will be lower if agents actually learn from others when the liquidity variance is large. However, this is not necessarily discouraging news. First, the proposition's implication holds only when the variance of liquidity is sufficiently large relative to other parameters of the economy, and social learning may actually yield high price volatility for a different specification of economic parameters. Second, the proposition tells that central nodes in the social network increase the information driven volatility component. This hints that high price volatility may be obtained in a network with central nodes. Using these clues, we construct an example in which social learning yields higher price volatility relative to social isolation:

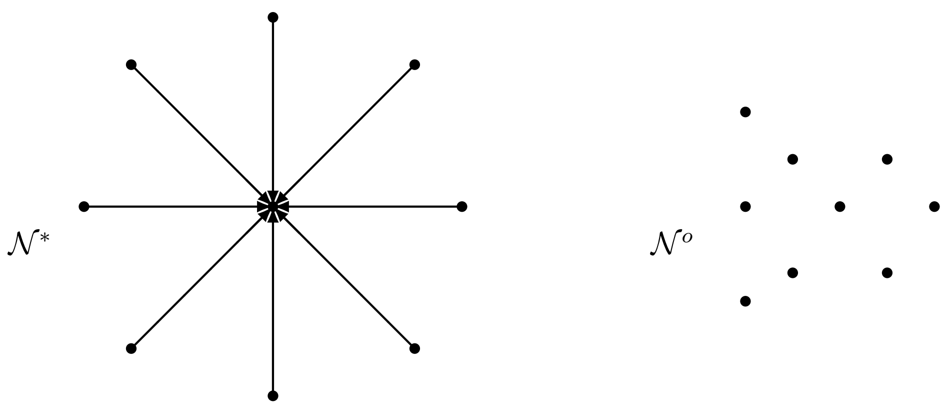

Figure 4: CENTRALIZATION VS. ISOLATION

In this figure, we see the illustrations of two social networks: a star , $\mathcal{N}^*$, in which all but one learn only from a unique agent, and a trivial network, $\mathcal{N}^o$, in which all agents are socially isolated.

Remark 1 *Let social network $\mathcal{N}^*$ be a star so that $\mathcal{S}(\mathcal{N}^*, 1) = \{1\}$ and $\mathcal{S}(\mathcal{N}^*, i) = \{1, i\}$, $i = 2, ..., n$. Also, let $\mathcal{N}^o$ be a trivial social network so that $\mathcal{S}(\mathcal{N}^o, i) = \{i\}$, $i = 1, ..., n$. Both $\mathcal{N}^*$ and $\mathcal{N}^o$ have unique linear equilibrium prices given any specification of the parameter vector $(\rho, \sigma_x^2, \sigma_\epsilon^2, \sigma_L^2, n) \in \mathbb{R}_{++}^4 \times \{2, 3, ...\}$.*

Suppose $\sigma_L^2 = s\, n^2$ for some $s \in \mathbb{R}_{++}$. Then

$$\text{var}\left(\tilde{p}^{\mathcal{N}^*}\right) \;>\; \text{var}\left(\tilde{p}^{\mathcal{N}^o}\right) \tag{3.4}$$

for sufficiently large number of agents n. Moreover, for any given constant $C > 0$ there exist

$N \in \mathbb{Z}_{++}$ *and* $\sigma_\epsilon^2(N) \in \mathbb{R}_{++}$ *such that for all* $n > N$ *and for all* $\sigma_\epsilon^2 < \sigma_\epsilon^2(N)$

$$\mathrm{var}\left(\tilde{p}^{\mathcal{N}^*}\right) \; > \; C \; \mathrm{var}\left(\tilde{p}^{\mathcal{N}^o}\right) \quad \textit{and} \quad \mathrm{var}\left(\tilde{p}^{\mathcal{N}^*}\right) \; > \; C \; \sigma_x^2. \tag{3.5}$$

Remark 1 considers an economy with large population size n and appropriately large liquidity variance σ_L^2. Under these restrictions, we see from (3.4) that price volatility in a star network is larger than that in a trivial network imposing complete social isolation. Hence, price-fundamental volatility ratio in the star network will always be higher than the ratio found under the assumption of social isolation. Also, (3.5) tells that price volatility in the star network can be made arbitrarily large relative to the volatility of risky payoff (i.e., the volatility of fundamental in our setup) by making private signals sufficiently precise. These results suggest an alternative explanation for the observed high volatility ratio of price to fundamentals: learning from common and central sources can be the cause.

3.5 Informational Efficiency and Social Networks

In this section, we investigate how different specifications of social networks affect agents' assessments of the fundamental, i.e., the risky payoff. In particular, we would like to see the implications of learning in social networks regarding informational efficiency. To that end, of course, we first need to explain what we mean by informational efficiency.

We define informational dominance for social networks as follows: *social network* $\mathcal{N}_1$ *is informationally dominated by social network* $\mathcal{N}_2$ *for the set of equilibrium prices* $\left\{\tilde{p}^{\mathcal{N}_1}, \tilde{p}^{\mathcal{N}_2}\right\}$ *if* $\mathrm{var}\left(\tilde{X}\middle|\tilde{\theta}_i, \mathrm{E}\left[\tilde{X}\middle|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N}_1,i)}\right], \tilde{p}^{\mathcal{N}_1}\right) \; \geq \; \mathrm{var}\left(\tilde{X}\middle|\tilde{\theta}_i, \mathrm{E}\left[\tilde{X}\middle|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N}_2,i)}\right], \tilde{p}^{\mathcal{N}_2}\right)$ *for all* $i = 1, ..., n$ *and* $\mathrm{var}\big(\tilde{X}\big|\tilde{\theta}_h, \mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N}_1,h)}\big], \tilde{p}^{\mathcal{N}_1}\big) > \mathrm{var}\big(\tilde{X}\big|\tilde{\theta}_h, \mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N}_2,h)}\big], \tilde{p}^{\mathcal{N}_2}\big)$ *for some* $h \in \{1, ..., n\}$.

This definition is sensible in the following sense. To determine informational dominance of a social network compared to another, we look at the precisions[8] of risky payoff $\tilde{X}$ conditional on agents' informations in each network. Naturally, the higher the precision, the better the assessment of risky payoff will be. Thus, if in network $\mathcal{N}_1$ all agents' informations regarding risky payoff is at least as precise as their informations would be in network $\mathcal{N}_2$ and one agent

[8] i.e., the reciprocal of variance.

in $\mathcal{N}_1$ has strictly more precise information regarding risky payoff than he would have in $\mathcal{N}_2$, then we can reasonably say that social network $\mathcal{N}_1$ informationally dominates social network $\mathcal{N}_2$. We should emphasize, though, that the informational dominance comparison between two social networks is dependent on the selection of equilibrium prices.

We next define informational efficiency for social networks: *social network $\hat{\mathcal{N}}$ is informationally efficient for the set of equilibrium prices $\left\{\tilde{p}^{\mathcal{N}} : \mathcal{N} \in \Omega\right\}$ if $\hat{\mathcal{N}}$ is not informationally dominated by any social network $\mathcal{N} \in \Omega$ for the equilibrium prices $\left\{\tilde{p}^{\hat{\mathcal{N}}}, \tilde{p}^{\mathcal{N}}\right\}$.*

The following proposition partially reveals the impact of social networks on informational efficiency:

Proposition 5 *Let $\{\tilde{p}^{\mathcal{N}} : \mathcal{N} \in \Omega\}$ be the set equilibrium prices that satisfy (3.1) and (3.2) for each $\mathcal{N} \in \Omega$. For sufficiently large liquidity variance σ_L^2,*

(a) social network $\mathcal{N}_1$ is informationally dominated by social network $\mathcal{N}_2$ for the set of equilibrium prices $\{\tilde{p}^{\mathcal{N}_1}, \tilde{p}^{\mathcal{N}_2}\}$ if $|\mathcal{S}(\mathcal{N}_1, i)| < |\mathcal{S}(\mathcal{N}_2), i|$ for all $i = 1, ..., n$,

(b) social network $\hat{\mathcal{N}}$ is informationally efficient for the set of equilibrium prices $\{\tilde{p}^{\mathcal{N}} : \mathcal{N} \in \Omega\}$ if $\mathcal{S}(\hat{\mathcal{N}}, i) = \{1, ..., n\}$ for all $i = 1, ..., n$.

Proposition 5 makes informational dominance comparison between social networks for the set of equilibrium prices satisfying (3.1)- (3.2) when the liquidity variance is sufficiently large. Part (a) of this proposition says that network $\mathcal{N}_1$ is informationally dominated by $\mathcal{N}_2$ if agents have fewer information sources in $\mathcal{N}_1$ than they would have in $\mathcal{N}_2$. Part (b) of the proposition reveals that the social network in which each agent learns from everyone else is informationally efficient.

Although, Proposition 5 seems to state the obvious at first sight, one has to be careful about the interaction between social inferences and price. The proposition holds only if the liquidity variance is sufficiently large which implies that social inferences are more reliable predictors of risky payoff than price is. In an economy with large liquidity variance, the higher the number of information sources, the better agents assess the risky payoff based on social inference, hence the results stated in Proposition 5 follow immediately. However, when liquidity variance is small, the number of each agent's information sources in social networks is not necessarily the sole factor in determination of informational dominance: under certain specifications of economic parameters, social inferences can impair information aggregation in price, which may, as a consequence, impair informational efficiency. The ways in which social inference can impair information aggregation in price have already been discussed in sections 3.2 and 3.4.

4 Appendix

We make extensive use of Projection Theorem in our proofs. Below is the statement of the theorem:

Projection Theorem: *Let $(\tilde{x}, \tilde{y})$ be an m-dimensional jointly normally distributed random vector with mean $\mu \in \mathbb{R}^m$ and variance-covariance matrix $\Sigma \in \mathbb{R}^{m \times m}$. Suppose that*

$$\mu = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}_{m \times 1} \quad \textit{and} \quad \Sigma = \begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{sx} & \Sigma_{yy} \end{pmatrix}_{m \times m},$$

where μ_a denotes the mean of random vector $\tilde{a} \in \{\tilde{x}, \tilde{y}\}$ and Σ_{ab} denotes the variance-covariance matrix of random vector $(\tilde{a}, \tilde{b}) \in \{\tilde{x}, \tilde{y}\} \times \{\tilde{x}, \tilde{y}\}$. Then the conditional distribution of $\tilde{x}$ given $\tilde{y} = y$ is normal with mean $\mu_x + \Sigma_{xy}\Sigma_{yy}^{-1}(y - \mu_y)$ and variance $\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx}$.

For $i = 1, ..., n$, let

$$w_{\mathcal{N},i} := \begin{cases} 1 & : \quad \mathcal{S}(\mathcal{N}, i) = \{i\} \\ 0 & : \quad \mathcal{S}(\mathcal{N}, i) \supsetneq \{i\}. \end{cases}$$

The following lemma provides the necessary and sufficient conditions for the existence of a linear equilibrium price.

Lemma 1 *For a given social network $\mathcal{N}$, a linear equilibrium price of the form*

$$\tilde{p} = P(\tilde{\theta}_1, ..., \tilde{\theta}_n, \tilde{L}) = \pi_0 + \sum_{i=1}^{n} \pi_i \, \tilde{\theta}_i - \gamma \, \tilde{L} \tag{4.1}$$

exists if and only if the following equations hold for some $(\pi_0, \pi_1, ..., \pi_n, \gamma) \in \mathbb{R}^{n+2}$*:*

$$\gamma = -\frac{1}{\sum_{i=1}^{n} \left(w_{\mathcal{N},i} \frac{a_{2i}}{b_i} + (1 - w_{\mathcal{N},i}) \frac{\hat{a}_{2i}}{\hat{b}_i + w_{\mathcal{N},i}} \right)}, \tag{4.2a}$$

$$\begin{aligned} \pi_i = \gamma \Bigg(& w_{\mathcal{N},i} \frac{a_{1i}}{b_i} + (1 - w_{\mathcal{N},i}) \frac{\hat{a}_{1i}}{\hat{b}_i + w_{\mathcal{N},i}} \\ & + \sum_{m=1}^{n} \frac{(1 - w_{\mathcal{N},m}) \hat{a}_{3m}}{\hat{b}_m + w_{\mathcal{N},m}} \frac{\big|\mathcal{S}(\mathcal{N}, m) \cap \{i\}\big| \sigma_x^2}{\sigma_\epsilon^2 + \big|\mathcal{S}(\mathcal{N}, m)\big| \sigma_x^2} \Bigg), \quad i = 1, ..., n, \end{aligned} \tag{4.2b}$$

$$\begin{aligned} \pi_0 = \gamma \sum_{i=1}^{n} \Bigg(& w_{\mathcal{N},i} \frac{a_{0i}}{b_i} + (1 - w_{\mathcal{N},i}) \frac{\hat{a}_{0i}}{\hat{b}_i + w_{\mathcal{N},i}} \\ & + (1 - w_{\mathcal{N},i}) \frac{\hat{a}_{3i}}{\hat{b}_i + w_{\mathcal{N},i}} \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \big|\mathcal{S}(\mathcal{N}, i)\big| \sigma_x^2} \mu_x \Bigg), \end{aligned} \tag{4.2c}$$

where

$$a_{0i} = -a_{2i} \pi_0 + \mu_x \left(b_i - a_{1i} - a_{2i} \sum_{j=1}^{n} \pi_j \right), \tag{4.3a}$$

$$a_{1i} = \sigma_x^2 \left(\sum_{j=1}^{n} \pi_j^2 \sigma_\epsilon^2 - \sum_{j=1}^{n} \pi_j \pi_i \sigma_\epsilon^2 + \gamma^2 \sigma_L^2 \right), \tag{4.3b}$$

$$\begin{aligned} a_{2i} = & -\gamma^2 \sigma_L^2 (\sigma_\epsilon^2 + \sigma_x^2) + \pi_i^2 \sigma_\epsilon^4 - \left(\pi_i - \sum_{j=1}^{n} \pi_j \right) \sigma_\epsilon^2 \sigma_x^2 + 2 \sum_{j=1}^{n} \pi_j \pi_i \sigma_\epsilon^2 \sigma_x^2 \\ & + \left(\sum_{j=1}^{n} \pi_j \right)^2 \sigma_x^4 - (\sigma_\epsilon^2 + \sigma_x^2) \left(\sum_{j=1}^{n} \pi_j^2 \sigma_\epsilon^2 + \left(\sum_{j=1}^{n} \pi_j \right)^2 \sigma_x^2 \right), \end{aligned} \tag{4.3c}$$

$$b_i = \rho \sigma_\epsilon^2 \sigma_x^2 \left(\sum_{j=1}^{n} \pi_j^2 \sigma_\epsilon^2 - \pi_i^2 \sigma_\epsilon^2 + \gamma^2 \sigma_L^2 \right). \tag{4.3d}$$

and

$$\hat{a}_{0i} = -\hat{a}_{2i}\,\pi_0 + \mu_x \left(\hat{b}_i - \hat{a}_{1i} - \hat{a}_{2i} \sum_{j=1}^{n} \pi_j - \hat{a}_{3i} \right), \tag{4.4a}$$

$$\hat{a}_{1i} = \sigma_\epsilon^2 \sigma_x^2 \left(\sum_{j=1}^{n} \pi_j - \sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k \right) \left(\sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k - |\mathcal{S}(\mathcal{N},i)| \pi_i \right), \tag{4.4b}$$

$$\begin{aligned} \hat{a}_{2i} = {} & \left(\sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k \right)^2 (\sigma_\epsilon^2 + \sigma_x^2)\sigma_\epsilon^2 - \gamma^2 (|\mathcal{S}(\mathcal{N},i)| - 1)(\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_L^2 \\ & -2\pi_i \sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k (\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)| \pi_i^2 (\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2 \\ & -(|\mathcal{S}(\mathcal{N},i)| - 1) \Bigg(\sum_{j=1}^{n} \pi_j^2 (\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2 + \sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k \sigma_x^2 \sigma_\epsilon^2 \\ & \qquad + \Big(\sum_{j=1}^{n} \pi_j \Big)^2 (\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_x^2 \\ & \qquad - \sum_{j=1}^{n} \pi_j \sigma_x^2 \Big(\sigma_\epsilon^2 + 2 \sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k \sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)| \sum_{j=1}^{n} \pi_j \sigma_x^2 \Big) \Bigg), \end{aligned} \tag{4.4c}$$

$$\begin{aligned} \hat{a}_{3i} = {} & \big(\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2 \big) \Bigg((|\mathcal{S}(\mathcal{N},i)| - 1)\big(\sigma_L^2 \gamma^2 + \sigma_\epsilon^2 \sum_{j=1}^{n} \pi_j^2 \big) - |\mathcal{S}(\mathcal{N},i)| \sigma_\epsilon^2 \pi_i^2 \\ & \qquad - \sigma_\epsilon^2 \sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k \sum_{j=1}^{n} \pi_j + \sigma_\epsilon^2 \pi_i \big(\sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k + \sum_{j=1}^{n} \pi_j \big) \Bigg), \end{aligned} \tag{4.4d}$$

$$\hat{b}_i = \rho\, \sigma_\epsilon^2 \sigma_x^2 \Bigg((|\mathcal{S}(\mathcal{N},i)| - 1) \Big(\sum_{j=1}^{n} \pi_j^2 \sigma_\epsilon^2 - \pi_i^2 \sigma_\epsilon^2 + \gamma^2 \sigma_L^2 \Big) - \Big(\sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k - \pi_i \Big)^2 \sigma_\epsilon^2 \Bigg). \tag{4.4e}$$

The equilibrium demands that correspond to price $\tilde{p}$ are of the form

$$\begin{aligned} z_i \left(\tilde{\theta}_i, \mathrm{E}\big[\tilde{X} \big| \{\tilde{\theta}_k\}_{k \in \mathcal{S}(\mathcal{N},i)}\big], \tilde{p} \right) = {} & w_{\mathcal{N},i} \frac{a_{0i}}{b_i} + (1 - w_{\mathcal{N},i}) \frac{\hat{a}_{0i}}{\hat{b}_i + w_{\mathcal{N},i}} \\ & + \left(w_{\mathcal{N},i} \frac{a_{1i}}{b_i} + (1 - w_{\mathcal{N},i}) \frac{\hat{a}_{1i}}{\hat{b}_i + w_{\mathcal{N},i}} \right) \tilde{\theta}_i \\ & + \left(w_{\mathcal{N},i} \frac{a_{2i}}{b_i} + (1 - w_{\mathcal{N},i}) \frac{\hat{a}_{2i}}{\hat{b}_i + w_{\mathcal{N},i}} \right) \tilde{p} \\ & + (1 - w_{\mathcal{N},i}) \frac{\hat{a}_{3i}}{\hat{b}_i + w_{\mathcal{N},i}} \mathrm{E}\big[\tilde{X} \big| \{\tilde{\theta}_k\}_{k \in \mathcal{S}(\mathcal{N},i)}\big], \quad i = 1, ..., n. \end{aligned}$$

Proof of Lemma 1: Given social network $\mathcal{N}$, suppose there exists a linear equilibrium price $\tilde{p}$ of the form (4.1). First, we solve for agents' demands at the equilibrium price. Fix an arbitrary $i \in \{1, ..., n\}$. Due to the CARA-normal setup, demand of agent i is given by

$$z_i\left(\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right) \;=\; \frac{\mathrm{E}\left[\tilde{X}\big|\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right] - p}{\rho\,\mathrm{var}\left(\tilde{X}\big|\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right)}. \tag{4.5}$$

We have two cases to analyze: (1) $\mathcal{S}(\mathcal{N}, i) = \{i\}$, and (2) $\mathcal{S}(\mathcal{N}, i) \supsetneq \{i\}$.

Case 1 $\big(\mathcal{S}(\mathcal{N}, i) = \{i\}\big)$*:* In this case, $\mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big] = \mathrm{E}[\tilde{X}|\theta_i]$. The information conveyed by $\mathrm{E}[\tilde{X}|\theta_i]$ is redundant for agent i since she knows θ_i. Thus, in this case,

$$\mathrm{E}\left[\tilde{X}\big|\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right] \;=\; \mathrm{E}\left[\tilde{X}\big|\theta_i, p\right], \tag{4.6a}$$

$$\mathrm{var}\left(\tilde{X}\big|\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right) \;=\; \mathrm{var}\left(\tilde{X}\big|\theta_i, p\right). \tag{4.6b}$$

Let V_i denote the variance-covariance matrix of $\left(\tilde{\theta}_i, \tilde{p}\right)$, and W_i denote the covariance matrix of $\tilde{X}$ and $\left(\tilde{\theta}_i, \tilde{p}\right)$. Since $\tilde{p}$ is a linear function of the normal random variables $\tilde{\theta}_1$, ..., $\tilde{\theta}_n$, $\tilde{L}$, the random vector $\left(\tilde{X}, \tilde{\theta}_i, \tilde{p}\right)$ is jointly normally distributed. Thus, due to the projection theorem, the conditional distribution of the risky payoff $\tilde{X}$, as assessed by agent i, has the mean

$$\mathrm{E}\left[\tilde{X}|\theta_i, p\right] \;=\; \mu_x + W_i' V_i^{-1} \begin{pmatrix} \theta_i - \mu_x \\ p - \pi_0 - \sum_{j=1}^n \pi_j\,\mu_x \end{pmatrix}, \tag{4.7a}$$

and the variance

$$\mathrm{var}\left(\tilde{X}|\theta_i, p\right) \;=\; \sigma_x^2 - W_i' V_i^{-1} W_i, \tag{4.7b}$$

where

$$V_i = \begin{pmatrix} {\sigma_x}^2 + {\sigma_\epsilon}^2 & \sum_{j=1}^n \pi_j\,{\sigma_x}^2 + \pi_i\,{\sigma_\epsilon}^2 \\ \sum_{j=1}^n \pi_j\,{\sigma_x}^2 + \pi_i\,{\sigma_\epsilon}^2 & \left(\sum_{j=1}^n \pi_j\right)^2 \sigma_x^2 + \sum_{j=1}^n \pi_j^2 \sigma_\epsilon^2 + \gamma^2 \sigma_L^2 \end{pmatrix},$$

$$W_i = \begin{pmatrix} \sigma_x^2 \\ \sum_{j=1}^n \pi_j\,\sigma_x^2 \end{pmatrix}.$$

Equations (4.5), (4.6a)-(4.6b) and (4.7a)- (4.7b) yield

$$z_i\left(\theta_i, \mathrm{E}\big[\tilde{X}\big|\theta_i\big], p\right) \;=\; \frac{a_{0i}}{b_i} + \frac{a_{1i}}{b_i}\,\theta_i + \frac{a_{2i}}{b_i}\,p, \tag{4.8}$$

where the coefficients a_{0i}, a_{1i}, a_{2i}, b_i satisfy (4.3a)-(4.3d).

Case 2 $\left(\mathcal{S}(\mathcal{N},i) \supsetneq \{i\}\right)$*:* Using the projection theorem, we obtain

$$\begin{aligned} \mathrm{E}\left[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right] &= \mu_x + \frac{1}{\frac{1}{\sigma_x^2} + \sum_{k\in\mathcal{S}(\mathcal{N},i)} \frac{1}{\sigma_\epsilon^2}} \sum_{k\in\mathcal{S}(\mathcal{N},i)} \frac{1}{\sigma_\epsilon^2}(\tilde{\theta}_k - \mu_x) \\ &= \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\,\mu_x + \frac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2} \sum_{k\in\mathcal{S}(\mathcal{N},i)} \tilde{\theta}_k. \qquad (4.9) \end{aligned}$$

Let $\hat{V}_i$ denote the variance-covariance matrix of $\left(\tilde{\theta}_i, \mathrm{E}\left[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], \tilde{p}\right)$. Also, let $\hat{W}_i$ denote the covariance matrix of risky payoff $\tilde{X}$ and $\left(\tilde{\theta}_i, \mathrm{E}\left[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], \tilde{p}\right)$. Since $\tilde{p}$ and $\mathrm{E}\left[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right]$ are linear functions of the normal random variables $\tilde{\theta}_1$, ..., $\tilde{\theta}_n$, $\tilde{L}$, the random vector $\left(\tilde{X}, \tilde{\theta}_i, \mathrm{E}\left[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], \tilde{p}\right)$ is jointly normally distributed. Once again, due to the projection theorem, the conditional distribution of the risky payoff $\tilde{X}$, as assessed by agent i, has the mean

$$\mathrm{E}\left[\tilde{X}\big|\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right] = \mu_x + \hat{W}_i'\hat{V}_i^{-1} \begin{pmatrix} \theta_i - \mu_x \\ \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big] - \mu_x \\ p - \pi_0 - \sum_{j=1}^n \pi_j\,\mu_x \end{pmatrix}, \qquad (4.10a)$$

and the variance

$$\mathrm{var}\left(\tilde{X}\big|\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right) = \sigma_x^2 - \hat{W}_i'\hat{V}_i^{-1}\hat{W}_i. \qquad (4.10b)$$

Note that

$$\hat{V}_i = \begin{pmatrix} {\sigma_x}^2 + {\sigma_\epsilon}^2 & \frac{|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\,\sigma_x^2 + \frac{\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\,\sigma_\epsilon^2 & \sum_{j=1}^n \pi_j\,{\sigma_x}^2 + \pi_i\,{\sigma_\epsilon}^2 \\ \frac{|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\,\sigma_x^2 + \frac{\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\,\sigma_\epsilon^2 & \mathrm{var}\left(\mathrm{E}\big[\tilde{X}|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big]\right) & \mathrm{cov}\left(\mathrm{E}\big[\tilde{X}|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big],\tilde{p}\right) \\ \sum_{j=1}^n \pi_j\,{\sigma_x}^2 + \pi_i\,{\sigma_\epsilon}^2 & \mathrm{cov}\left(\mathrm{E}\big[\tilde{X}|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big],\tilde{p}\right) & \mathrm{var}(\tilde{p}) \end{pmatrix},$$

where

$$\begin{aligned} \mathrm{cov}\left(\mathrm{E}\left[\tilde{X}|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right], \tilde{p}\right) &= \frac{|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2} \sum_{j=1}^n \pi_j\,\sigma_x^2 + \frac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2} \sum_{k\in\mathcal{S}(\mathcal{N},i)} \pi_k\,\sigma_\epsilon^2, \\ \mathrm{var}\left(\mathrm{E}\left[\tilde{X}|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right]\right) &= \left(\frac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\right)^2 \left(|\mathcal{S}(\mathcal{N},i)|^2\sigma_x^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_\epsilon^2\right), \\ \mathrm{var}(\tilde{p}) &= \left(\sum_{j=1}^n \pi_j\right)^2 \sigma_x^2 + \sum_{j=1}^n \pi_j^2\sigma_\epsilon^2 + \gamma^2\sigma_L^2, \end{aligned}$$

and

$$\hat{W}_i = \begin{pmatrix} \sigma_x^2 \\ \frac{|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\,\sigma_x^2 \\ \sum_{j=1}^n \pi_j\,\sigma_x^2 \end{pmatrix}.$$

Also, note that $\hat{V}_i$ is invertible since $\mathcal{S}(\mathcal{N},i) \supsetneq \{i\}$.[9] Going back to equations (4.5), (4.10a) and (4.10b), we derive

$$z_i\left(\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right) = \frac{\hat{a}_{0i}}{\hat{b}_i} + \frac{\hat{a}_{1i}}{\hat{b}_i}\,\theta_i + \frac{\hat{a}_{2i}}{\hat{b}_i}\,p + \frac{\hat{a}_{3i}}{\hat{b}_i}\,\mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], \tag{4.11}$$

where the coefficients $\hat{a}_{0i}$, $\hat{a}_{1i}$, $\hat{a}_{2i}$, $\hat{a}_{3i}$, $\hat{b}_i$ satisfy (4.4a)-(4.4e).

Combining the demands for cases (1) and (2), namely (4.8) and (4.11), we obtain[10]

$$\begin{aligned} z_i\left(\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right) &= w_{\mathcal{N},i}\,\frac{a_{0i}}{b_i} + (1-w_{\mathcal{N},i})\,\frac{\hat{a}_{0i}}{\hat{b}_i + w_{\mathcal{N},i}} \\ &+ \left(w_{\mathcal{N},i}\,\frac{a_{1i}}{b_i} + (1-w_{\mathcal{N},i})\,\frac{\hat{a}_{1i}}{\hat{b}_i + w_{\mathcal{N},i}}\right)\theta_i \\ &+ \left(w_{\mathcal{N},i}\,\frac{a_{2i}}{b_i} + (1-w_{\mathcal{N},i})\,\frac{\hat{a}_{2i}}{\hat{b}_i + w_{\mathcal{N},i}}\right)p \\ &+ (1-w_{\mathcal{N},i})\,\frac{\hat{a}_{3i}}{\hat{b}_i + w_{\mathcal{N},i}}\,\mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big]. \end{aligned} \tag{4.12}$$

Since we initially fixed an arbitrary $i \in \{1, ..., n\}$, (4.12) holds for all $i = 1, ..., n$.

Next, we impose the market-clearing condition on demands given by (4.12):

$$\sum_{i=1}^n z_i\left(\theta_i, \mathrm{E}\big[\tilde{X}\big|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], p\right) = L. \tag{4.13}$$

[9]If $\mathcal{S}(\mathcal{N},i)$ only consisted of i, $\hat{V}_i$ would not have full rank since the second row of $\hat{V}_i$ (i.e., the covariance of $\mathrm{E}\big[\tilde{X}\big|\theta_i\big]$ and $\left(\theta_i, \mathrm{E}\big[\tilde{X}\big|\theta_i\big], p\right)$) would be a multiple of the first row of $\hat{V}_i$ (i.e., the covariance of θ_i and $\left(\theta_i, \mathrm{E}\big[\tilde{X}\big|\theta_i\big], p\right)$). This simply follows from the fact that the information conveyed by $\mathrm{E}\big[\tilde{X}\big|\theta_i\big]$ is redundant for agent i because she knows θ_i.

[10]We add $w_{\mathcal{N},i}$ to $\hat{b}_i$ in order to avoid division by zero when $\mathcal{S}(\mathcal{N},i) = \{i\}$. After this modification, demand given by (4.12) is well-defined, and still consistent with demands given by (4.8) and (4.11).

Solving for p from (4.13), we obtain

$$p = \frac{\sum_{i=1}^{n}\left(w_{\mathcal{N},i}\,\frac{a_{0i}}{b_i}+(1-w_{\mathcal{N},i})\,\frac{\hat{a}_{0i}}{\hat{b}_i+w_{\mathcal{N},i}}\right)+\sum_{i=1}^{n}\left(w_{\mathcal{N},i}\,\frac{a_{1i}}{b_i}+(1-w_{\mathcal{N},i})\,\frac{\hat{a}_{1i}}{\hat{b}_i+w_{\mathcal{N},i}}\right)\theta_i}{-\sum_{i=1}^{n}\left(w_{\mathcal{N},i}\,\frac{a_{2i}}{b_i}+(1-w_{\mathcal{N},i})\,\frac{\hat{a}_{2i}}{\hat{b}_i+w_{\mathcal{N},i}}\right)} + \frac{\sum_{i=1}^{n}(1-w_{\mathcal{N},i})\,\frac{\hat{a}_{3i}}{\hat{b}_i+w_{\mathcal{N},i}}\mathrm{E}\left[\tilde{X}|\{\theta_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right]-L}{-\sum_{i=1}^{n}\left(w_{\mathcal{N},i}\,\frac{a_{2i}}{b_i}+(1-w_{\mathcal{N},i})\,\frac{\hat{a}_{2i}}{\hat{b}_i+w_{\mathcal{N},i}}\right)}. \tag{4.14}$$

Consequently, (4.1), (4.9) and (4.14) together imply the system of equations (4.2a)-(4.2c).

Therefore, a linear equilibrium price of the form (4.1) exists if and only if equations (4.3a)-(4.3d), (4.4a)-(4.4e) and (4.2a)-(4.2c) hold for some $(\pi_0, \pi_1, ..., \pi_n, \gamma) \in \mathbb{R}^{n+2}$. □

Proof of Proposition 1: Following Lemma 1, if $\tilde{p} = \pi_0 + \sum_{i=1}^{n} \pi_i\,\tilde{\theta}_i - \gamma\,\tilde{L}$ is a linear equilibrium price for social network $\mathcal{N}$, then for all $i = 1, ..., n$

$$\begin{aligned}\pi_i = \gamma\Bigg(&\frac{w_{\mathcal{N},i}\,\left(\sum_{j=1}^{n}\pi_j^2\sigma_\epsilon^2-\sum_{j=1}^{n}\pi_j\pi_i\sigma_\epsilon^2+\gamma^2\sigma_L^2\right)}{\rho\,\sigma_\epsilon^2\left(\sum_{j=1}^{n}\pi_j^2\sigma_\epsilon^2-\pi_i^2\sigma_\epsilon^2+\gamma^2\sigma_L^2\right)}\\
&+\frac{(1-w_{\mathcal{N},i})\,\sigma_\epsilon^2\sigma_x^2\left(\sum_{j=1}^{n}\pi_j-\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k\right)\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k-|\mathcal{S}(\mathcal{N},i)|\pi_i\right)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^{n}\pi_j^2\sigma_\epsilon^2-\pi_i^2\sigma_\epsilon^2+\gamma^2\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k-\pi_i\right)^2\sigma_\epsilon^2\right)+w_{\mathcal{N},i}}\\
&+\sum_{m=1}^{n}(1-w_{\mathcal{N},m})\,|\mathcal{S}(\mathcal{N},m)\cap\{i\}|\sigma_x^2\,\times\\
&\Bigg[\frac{(|\mathcal{S}(\mathcal{N},m)|-1)\big(\sigma_L^2\gamma^2+\sigma_\epsilon^2\sum_{j=1}^{n}\pi_j^2\big)-|\mathcal{S}(\mathcal{N},m)|\sigma_\epsilon^2\pi_m^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^{n}\pi_j^2\sigma_\epsilon^2-\pi_m^2\sigma_\epsilon^2+\gamma^2\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},m)}\pi_k-\pi_m\right)^2\sigma_\epsilon^2\right)+w_{\mathcal{N},m}}\\
&+\frac{-\sigma_\epsilon^2\sum_{k\in\mathcal{S}(\mathcal{N},m)}\pi_k\sum_{j=1}^{n}\pi_j+\sigma_\epsilon^2\pi_i\big(\sum_{k\in\mathcal{S}(\mathcal{N},m)}\pi_k+\sum_{j=1}^{n}\pi_j\big)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^{n}\pi_j^2\sigma_\epsilon^2-\pi_m^2\sigma_\epsilon^2+\gamma^2\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},m)}\pi_k-\pi_m\right)^2\sigma_\epsilon^2\right)+w_{\mathcal{N},m}}\Bigg]\Bigg).\end{aligned}$$

Let $q_i := \frac{\pi_i}{\gamma}$, $i = 1, ..., n$. Then, the equation above yields

$$
\begin{aligned}
q_i \;=\; & \frac{w_{\mathcal{N},i}\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - \sum_{j=1}^n q_j q_i \sigma_\epsilon^2 + \sigma_L^2\right)}{\rho\,\sigma_\epsilon^2\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - q_i^2\sigma_\epsilon^2 + \sigma_L^2\right)} \\
& + \frac{(1-w_{\mathcal{N},i})\,\sigma_\epsilon^2\sigma_x^2\left(\sum_{j=1}^n q_j - \sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k\right)\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k - |\mathcal{S}(\mathcal{N},i)|q_i\right)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - q_i^2\sigma_\epsilon^2 + \sigma_L^2\right) - \left(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k - q_i\right)^2\sigma_\epsilon^2\right) + w_{\mathcal{N},i}} \\
& + \sum_{m=1}^n (1-w_{\mathcal{N},m})\,|\mathcal{S}(\mathcal{N},m)\cap\{i\}|\sigma_x^2 \,\times \\
& \left[\frac{(|\mathcal{S}(\mathcal{N},m)|-1)\big(\sigma_L^2 + \sigma_\epsilon^2\sum_{j=1}^n q_j^2\big) - |\mathcal{S}(\mathcal{N},m)|\sigma_\epsilon^2 q_m^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - q_m^2\sigma_\epsilon^2 + \sigma_L^2\right) - \left(\sum_{k\in\mathcal{S}(\mathcal{N},m)} q_k - q_m\right)^2\sigma_\epsilon^2\right) + w_{\mathcal{N},m}}\right. \\
& \left. + \frac{-\sigma_\epsilon^2\sum_{k\in\mathcal{S}(\mathcal{N},m)} q_k \sum_{j=1}^n q_j + \sigma_\epsilon^2 q_i\big(\sum_{k\in\mathcal{S}(\mathcal{N},m)} q_k + \sum_{j=1}^n q_j\big)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - q_m^2\sigma_\epsilon^2 + \sigma_L^2\right) - \left(\sum_{k\in\mathcal{S}(\mathcal{N},m)} q_k - q_m\right)^2\sigma_\epsilon^2\right) + w_{\mathcal{N},m}}\right].
\end{aligned}
\tag{4.15}
$$

Note that equation (4.15) is independent from γ and π_0. Next we show that (4.15) actually holds for some $(q_1, ..., q_n) \in \mathbb{R}^n$. Consider the function $f : \mathbb{R}^n \to \mathbb{R}^n$ satisfying[11]

$$
\begin{aligned}
(f(q_1, ..., q_n))_i \;=\; & \frac{w_{\mathcal{N},i}\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - \sum_{j=1}^n q_j q_i \sigma_\epsilon^2 + \sigma_L^2\right)}{\rho\,\sigma_\epsilon^2\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - q_i^2\sigma_\epsilon^2 + \sigma_L^2\right)} \\
& + \frac{(1-w_{\mathcal{N},i})\,\sigma_\epsilon^2\sigma_x^2\left(\sum_{j=1}^n q_j - \sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k\right)\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k - |\mathcal{S}(\mathcal{N},i)|q_i\right)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - q_i^2\sigma_\epsilon^2 + \sigma_L^2\right) - \left(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k - q_i\right)^2\sigma_\epsilon^2\right) + w_{\mathcal{N},i}} \\
& + \sum_{m=1}^n (1-w_{\mathcal{N},m})\,|\mathcal{S}(\mathcal{N},m)\cap\{i\}|\sigma_x^2 \,\times \\
& \left[\frac{(|\mathcal{S}(\mathcal{N},m)|-1)\big(\sigma_L^2 + \sigma_\epsilon^2\sum_{j=1}^n q_j^2\big) - |\mathcal{S}(\mathcal{N},m)|\sigma_\epsilon^2 q_m^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - q_m^2\sigma_\epsilon^2 + \sigma_L^2\right) - \left(\sum_{k\in\mathcal{S}(\mathcal{N},m)} q_k - q_m\right)^2\sigma_\epsilon^2\right) + w_{\mathcal{N},m}}\right. \\
& \left. + \frac{-\sigma_\epsilon^2\sum_{k\in\mathcal{S}(\mathcal{N},m)} q_k \sum_{j=1}^n q_j + \sigma_\epsilon^2 q_i\big(\sum_{k\in\mathcal{S}(\mathcal{N},m)} q_k + \sum_{j=1}^n q_j\big)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^n q_j^2\sigma_\epsilon^2 - q_m^2\sigma_\epsilon^2 + \sigma_L^2\right) - \left(\sum_{k\in\mathcal{S}(\mathcal{N},m)} q_k - q_m\right)^2\sigma_\epsilon^2\right) + w_{\mathcal{N},m}}\right].
\end{aligned}
$$

[11] $(f(q_1, ..., q_n))_i$ denotes the ith component of the vector value of $f(q_1, ..., q_n)$.

There exists a real vector $(q_1^*, ..., q_n^*)$ satisfying (4.15) if and only if $(q_1^*, ..., q_n^*)$ is a fixed point of f. Using Brouwer's Theorem, we will show that f has a fixed point for sufficiently large σ_L^2. It is easy to check that f is continuous in the domain $\left[0, 2+\frac{n}{\rho\sigma_\epsilon^2}\right]^n$. Also, if $(q_1, ..., q_n) \in \left[0, 2+\frac{n}{\rho\sigma_\epsilon^2}\right]^n$, then for all $i = 1, ..., n$

$$\lim_{\sigma_L^2\to\infty} (f(q_1, ..., q_n))_i \;\; = \;\; w_{\mathcal{N},i} + \sum_{m=1}^{n}(1-w_{\mathcal{N},m})|\mathcal{S}(\mathcal{N},m)\cap\{i\}|\frac{1}{\rho\sigma_\epsilon^2}.$$

Thus, for sufficiently large σ_L^2, if $(q_1, ..., q_n) \in \left[0, 2+\frac{n}{\rho\sigma_\epsilon^2}\right]^n$, then

$$0 < (f(q_1, ..., q_n))_i < 2 + \frac{n}{\rho\sigma_\epsilon^2}, \qquad \forall i = 1, ..., n.$$

This proves that $f\left(\left[0, 2+\frac{n}{\rho\sigma_\epsilon^2}\right]^n\right) \subseteq \left(0, 2+\frac{n}{\rho\sigma_\epsilon^2}\right)^n$ for sufficiently large σ_L^2. Then from Brouwer's Theorem, there exists $(q_1^*, ..., q_n^*) \in \left(0, 2+\frac{n}{\rho\sigma_\epsilon^2}\right)^n$ such that

$$f(q_1^*, ..., q_n^*) \;\; = \;\; (q_1^*, ..., q_n^*)$$

when σ_L^2 is sufficiently large. Consequently, equation (4.15) holds for $(q_1^*, ..., q_n^*) \in \left(0, 2+\frac{n}{\rho\sigma_\epsilon^2}\right)^n$ for sufficiently large σ_L^2.

Let

$$\begin{aligned}
\gamma^{\mathcal{N}} \;\; = \;\; & \Bigg(1+\sum_{i=1}^n \Bigg[\frac{w_{\mathcal{N},i}\Big(\sum_{j=1}^n q_j^* - q_i^*\Big)\sigma_\epsilon^2\sigma_x^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\Big(\sum_{j=1}^n (q_j^*)^2\sigma_\epsilon^2-(q_i^*)^2\sigma_\epsilon^2+\sigma_L^2\Big)} \\
& \quad + \frac{(1-w_{\mathcal{N},i})(|\mathcal{S}(\mathcal{N},i)|-1)\Big(\sum_{j=1}^n q_j^* - \sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k^*\Big)\sigma_\epsilon^2\sigma_x^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\Bigg((|\mathcal{S}(\mathcal{N},i)|-1)\Big(\sum_{j=1}^n (q_j^*)^2\sigma_\epsilon^2-(q_i^*)^2\sigma_\epsilon^2+\sigma_L^2\Big)-\Big(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k^*-q_i^*\Big)^2\sigma_\epsilon^2\Bigg)+w_{\mathcal{N},i}}\Bigg]\Bigg) \\
& \Bigg(-\sum_{i=1}^n \Bigg[w_{\mathcal{N},i}\frac{-\sigma_L^2(\sigma_\epsilon^2+\sigma_x^2)+(q_i^*)^2\sigma_\epsilon^4+2\sum_{j=1}^n q_j^* q_i^*\sigma_\epsilon^2\sigma_x^2+\Big(\sum_{j=1}^n q_j^*\Big)^2\sigma_x^4-(\sigma_\epsilon^2+\sigma_x^2)\Big(\sum_{j=1}^n (q_j^*)^2\sigma_\epsilon^2+\Big(\sum_{j=1}^n q_j^*\Big)^2\sigma_x^2\Big)}{\rho\,\sigma_\epsilon^2\sigma_x^2\Big(\sum_{j=1}^n (q_j^*)^2\sigma_\epsilon^2-(q_i^*)^2\sigma_\epsilon^2+\sigma_L^2\Big)} \\
& \quad +(1-w_{\mathcal{N},i})\Bigg(\frac{\Big(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k^*\Big)^2(\sigma_\epsilon^2+\sigma_x^2)\sigma_\epsilon^2-(|\mathcal{S}(\mathcal{N},i)|-1)(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_L^2-2q_i^*\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k^*(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\Big((|\mathcal{S}(\mathcal{N},i)|-1)\Big(\sum_{j=1}^n (q_j^*)^2\sigma_\epsilon^2-(q_i^*)^2\sigma_\epsilon^2+\sigma_L^2\Big)-\Big(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k^*-q_i^*\Big)^2\sigma_\epsilon^2\Big)+w_{\mathcal{N},i}} \\
& \quad \frac{+|\mathcal{S}(\mathcal{N},i)|(q_i^*)^2(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2-(|\mathcal{S}(\mathcal{N},i)|-1)\sum_{j=1}^n (q_j^*)^2(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\Big((|\mathcal{S}(\mathcal{N},i)|-1)\Big(\sum_{j=1}^n (q_j^*)^2\sigma_\epsilon^2-(q_i^*)^2\sigma_\epsilon^2+\sigma_L^2\Big)-\Big(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k^*-q_i^*\Big)^2\sigma_\epsilon^2\Big)+w_{\mathcal{N},i}} \\
& \quad -\frac{(|\mathcal{S}(\mathcal{N},i)|-1)\Big(\Big(\sum_{j=1}^n q_j^*\Big)^2(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_x^2-\sum_{j=1}^n q_j^*\sigma_x^2\Big(2\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k^*\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sum_{j=1}^n q_j^*\sigma_x^2\Big)\Big)}{\rho\,\sigma_\epsilon^2\sigma_x^2\Big((|\mathcal{S}(\mathcal{N},i)|-1)\Big(\sum_{j=1}^n (q_j^*)^2\sigma_\epsilon^2-(q_i^*)^2\sigma_\epsilon^2+\sigma_L^2\Big)-\Big(\sum_{k\in\mathcal{S}(\mathcal{N},i)} q_k^*-q_i^*\Big)^2\sigma_\epsilon^2\Big)+w_{\mathcal{N},i}}\Bigg)\Bigg]\Bigg)^{-1}, \\
\pi_i^{\mathcal{N}} \;\; = \;\; & \gamma^{\mathcal{N}} q_i^*, \quad i = 1, ..., n.
\end{aligned}$$

The coefficients $\gamma^{\mathcal{N}}$ and $\pi_i^{\mathcal{N}}$, $i = 1, ..., n$, satisfy the equations in Lemma 1 for sufficiently large σ_L^2. Using $\gamma^{\mathcal{N}}$ and $\pi_i^{\mathcal{N}}$, $i = 1, ..., n$, it is also straightforward to find $\pi_0^{\mathcal{N}}$ that satisfies the equations in Lemma 1, however the calculation is tedious. Following Lemma 1,

$$\tilde{p}^{\mathcal{N}} \;=\; \pi_0^{\mathcal{N}} + \sum_{i=1}^{n} \pi_i^{\mathcal{N}} \tilde{\theta}_i - \gamma^{\mathcal{N}} \tilde{L}$$

is a linear equilibrium price when σ_L^2 is sufficiently large. Moreover,

$$\lim_{\sigma_L^2 \to \infty} \gamma^{\mathcal{N}} = \frac{\rho\, \sigma_\epsilon^2\, \sigma_x^2}{\sum_{i=1}^{n} \left(w_{\mathcal{N},i}\, (\sigma_\epsilon^2 + \sigma_x^2) + (1 - w_{\mathcal{N},i})\, (\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N}, i)| \sigma_x^2)\right)}$$

since $(q_1^*, ..., q_n^*) \in \left(0, 2 + \frac{n}{\rho\, \sigma_\epsilon^2}\right)^n$. Therefore,

$$0 < \lim_{\sigma_L^2 \to \infty} \gamma^{\mathcal{N}} < \infty, \qquad 0 < \lim_{\sigma_L^2 \to \infty} \pi_i^{\mathcal{N}} = \lim_{\sigma_L^2 \to \infty} \gamma^{\mathcal{N}} \left(2 + \frac{n}{\rho\, \sigma_\epsilon^2}\right) < \infty, \quad i = 1, ..., n. \quad \square$$

Proof of Proposition 2: Since $\tilde{p}^{\mathcal{N}}$ is a linear equilibrium price, it immediately follows from Lemma 1 that

$$\begin{aligned}
\pi_i^{\mathcal{N}} \;=\; \gamma^{\mathcal{N}} \Bigg(& \frac{\sigma_\epsilon^2 \sigma_x^2 \left(\sum_{j=1}^{n} \pi_j^{\mathcal{N}} - \sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k^{\mathcal{N}}\right)\left(\sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k^{\mathcal{N}} - |\mathcal{S}(\mathcal{N},i)| \pi_i^{\mathcal{N}}\right)}{\rho\, \sigma_\epsilon^2 \sigma_x^2 \left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^{n} (\pi_j^{\mathcal{N}})^2 \sigma_\epsilon^2 - (\pi_i^{\mathcal{N}})^2 \sigma_\epsilon^2 + (\gamma^{\mathcal{N}})^2 \sigma_L^2\right) - \left(\sum_{k \in \mathcal{S}(\mathcal{N},i)} \pi_k^{\mathcal{N}} - \pi_i^{\mathcal{N}}\right)^2 \sigma_\epsilon^2\right)} \\
& + \sum_{m=1}^{n} (1 - w_{\mathcal{N},m})\, |\mathcal{S}(\mathcal{N},m) \cap \{i\}| \sigma_x^2 \times \\
& \Bigg[\frac{(|\mathcal{S}(\mathcal{N},m)|-1)\left(\sigma_L^2 (\gamma^{\mathcal{N}})^2 + \sigma_\epsilon^2 \sum_{j=1}^{n} (\pi_j^{\mathcal{N}})^2\right) - |\mathcal{S}(\mathcal{N},m)| \sigma_\epsilon^2 (\pi_m^{\mathcal{N}})^2 - \sigma_\epsilon^2 \sum_{k \in \mathcal{S}(\mathcal{N},m)} \pi_k^{\mathcal{N}} \sum_{j=1}^{n} \pi_j^{\mathcal{N}}}{\rho\, \sigma_\epsilon^2 \sigma_x^2 \left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^{n} (\pi_j^{\mathcal{N}})^2 \sigma_\epsilon^2 - (\pi_m^{\mathcal{N}})^2 \sigma_\epsilon^2 + (\gamma^{\mathcal{N}})^2 \sigma_L^2\right) - \left(\sum_{k \in \mathcal{S}(\mathcal{N},m)} \pi_k^{\mathcal{N}} - \pi_m^{\mathcal{N}}\right)^2 \sigma_\epsilon^2\right) + w_{\mathcal{N},m}} \\
& + \frac{\sigma_\epsilon^2 \pi_i^{\mathcal{N}} \left(\sum_{k \in \mathcal{S}(\mathcal{N},m)} \pi_k^{\mathcal{N}} + \sum_{j=1}^{n} \pi_j^{\mathcal{N}}\right)}{\rho\, \sigma_\epsilon^2 \sigma_x^2 \left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^{n} (\pi_j^{\mathcal{N}})^2 \sigma_\epsilon^2 - (\pi_m^{\mathcal{N}})^2 \sigma_\epsilon^2 + (\gamma^{\mathcal{N}})^2 \sigma_L^2\right) - \left(\sum_{k \in \mathcal{S}(\mathcal{N},m)} \pi_k^{\mathcal{N}} - \pi_m^{\mathcal{N}}\right)^2 \sigma_\epsilon^2\right) + w_{\mathcal{N},m}} \Bigg] \Bigg)
\end{aligned}$$

for all i that satisfies $\mathcal{S}(\mathcal{N}, i) \supsetneq \{i\}$. Let $\{i, i'\} \subset \{1, ..., n\}$ such that $\mathcal{S}(\mathcal{N}, i) \supsetneq \{i\}$ and $\mathcal{S}(\mathcal{N}, i') \supsetneq \{i'\}$. Given that $0 < \lim_{\sigma_L^2 \to \infty} \gamma^{\mathcal{N}} < \infty$ and $0 < \lim_{\sigma_L^2 \to \infty} \pi_i^{\mathcal{N}} < \infty$, $i = 1, ..., n$,

$$\begin{aligned}
\lim_{\sigma_L^2 \to \infty} \left(\pi_i^{\mathcal{N}} - \pi_{i'}^{\mathcal{N}}\right) \;&=\; \frac{\lim_{\sigma_L^2 \to \infty} \gamma^{\mathcal{N}}}{\rho\, \sigma_\epsilon^2} \sum_{m=1}^{n} (1 - w_{\mathcal{N},m})\, (|\mathcal{S}(\mathcal{N}, m) \cap \{i\}| - |\mathcal{S}(\mathcal{N}, m) \cap \{i'\}|) \\
&=\; \frac{\lim_{\sigma_L^2 \to \infty} \gamma^{\mathcal{N}}}{\rho\, \sigma_\epsilon^2} \sum_{m=1}^{n} (|\mathcal{S}(\mathcal{N}, m) \cap \{i\}| - |\mathcal{S}(\mathcal{N}, m) \cap \{i'\}|)\,.
\end{aligned}$$

Therefore, if $\sum_{m=1}^{n} |\mathcal{S}(\mathcal{N}, m) \cap \{i\}| > \sum_{m=1}^{n} |\mathcal{S}(\mathcal{N}, m) \cap \{i'\}|$, then $\pi_i^{\mathcal{N}} > \pi_{i'}^{\mathcal{N}}$ for sufficiently large σ_l^2. $\square$

Proof of Proposition 3: We will prove this proposition in three steps:

Step 1: Following Lemma 1, if $|\mathcal{S}(\mathcal{N},i)| > 1$, then equilibrium demand of agent i that corresponds to $\tilde{p}^{\mathcal{N}}$ is of the form

$$\tilde{z}_i^{\mathcal{N}} \;:=\; z_i\left(\tilde{\theta}_i, \mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big], \tilde{p}^{\mathcal{N}}\right) \;=\; \frac{\hat{a}_{0i}}{\hat{b}_i} + \frac{\hat{a}_{1i}}{\hat{b}_i}\,\tilde{\theta}_i + \frac{\hat{a}_{2i}}{\hat{b}_i}\,\tilde{p}^{\mathcal{N}} + \frac{\hat{a}_{3i}}{\hat{b}_i}\,\mathrm{E}\big[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\big],$$

where the coefficients $\hat{a}_{0i}$, $\hat{a}_{1i}$, $\hat{a}_{2i}$, $\hat{a}_{3i}$, $\hat{b}_i$ are as given in Lemma 1. The projection theorem implies that

$$\mathrm{E}\left[\tilde{X}\big|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}\right] \;=\; \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\,\mu_x + \frac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\sum_{k\in\mathcal{S}(\mathcal{N},i)}\tilde{\theta}_k.$$

Thus, for $i = 1, ..., n$ with $|\mathcal{S}(\mathcal{N},i)| > 1$

$$\begin{aligned}\mathrm{var}(\tilde{z}_i^{\mathcal{N}}) \;=\; & \left(\tfrac{\hat{a}_{1i}}{\hat{b}_i} + \textstyle\sum_{k\in\mathcal{S}(\mathcal{N},i)}\left(\tfrac{\hat{a}_{2i}}{\hat{b}_i}\pi_k^{\mathcal{N}} + \tfrac{\hat{a}_{3i}}{\hat{b}_i}\tfrac{\sigma_x^2}{\sigma_x^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_\epsilon^2}\right) + \textstyle\sum_{k\notin\mathcal{S}(\mathcal{N},i)}\tfrac{\hat{a}_{2i}}{\hat{b}_i}\pi_k^{\mathcal{N}}\right)^2\sigma_x^2 \\ & + \left(\textstyle\sum_{k\in\mathcal{S}(\mathcal{N},i)\setminus\{i\}}\left(\tfrac{\hat{a}_{2i}}{\hat{b}_i}\pi_k^{\mathcal{N}} + \tfrac{\hat{a}_{3i}}{\hat{b}_i}\tfrac{\sigma_x^2}{\sigma_x^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_\epsilon^2}\right)^2 + \textstyle\sum_{k\notin\mathcal{S}(\mathcal{N},i)}\left(\tfrac{\hat{a}_{2i}}{\hat{b}_i}\pi_k^{\mathcal{N}}\right)^2\right)\sigma_\epsilon^2 + \left(\tfrac{\hat{a}_{2i}}{\hat{b}_i}\gamma^{\mathcal{N}}\right)^2\sigma_L^2,\end{aligned} \tag{4.16}$$

and for $i \neq h$ with $|\mathcal{S}(\mathcal{N},i)| > 1$, $|\mathcal{S}(\mathcal{N},h)| > 1$,

$$\begin{aligned}\mathrm{cov}(\tilde{z}_i^{\mathcal{N}}, \tilde{z}_h^{\mathcal{N}}) \;=\; & \left(\tfrac{\hat{a}_{1i}}{\hat{b}_i} + \tfrac{\hat{a}_{2i}}{\hat{b}_i}\textstyle\sum_{j=1}^n\pi_j^{\mathcal{N}} + \tfrac{\hat{a}_{3i}}{\hat{b}_i}\tfrac{|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\right)\left(\tfrac{\hat{a}_{1h}}{\hat{b}_h} + \tfrac{\hat{a}_{2h}}{\hat{b}_h}\textstyle\sum_{j=1}^n\pi_j^{\mathcal{N}} + \tfrac{\hat{a}_{3h}}{\hat{b}_h}\tfrac{|\mathcal{S}(\mathcal{N},h)|\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},h)|\sigma_x^2}\right)\sigma_x^2 \\ & + \tfrac{\hat{a}_{2i}}{\hat{b}_i}\tfrac{\hat{a}_{2h}}{\hat{b}_h}(\gamma^{\mathcal{N}})^2\sigma_L^2 + \tfrac{\hat{a}_{1i}}{\hat{b}_i}\left(\tfrac{\hat{a}_{1h}}{\hat{b}_h} + \tfrac{\hat{a}_{2h}}{\hat{b}_h}\pi_h^{\mathcal{N}} + \tfrac{\hat{a}_{3h}}{\hat{b}_h}\tfrac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},h)|\sigma_x^2}\right)\sigma_\epsilon^2 \\ & + \tfrac{\hat{a}_{2i}}{\hat{b}_i}\left(\tfrac{\hat{a}_{1h}}{\hat{b}_h}\pi_h^{\mathcal{N}} + \tfrac{\hat{a}_{2h}}{\hat{b}_h}\textstyle\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2 + \tfrac{\hat{a}_{3h}}{\hat{b}_h}\tfrac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},h)|\sigma_x^2}\textstyle\sum_{k\in\mathcal{S}(\mathcal{N},h)}\pi_k^{\mathcal{N}}\right)\sigma_\epsilon^2 \\ & + \tfrac{\hat{a}_{3h}}{\hat{b}_h}\tfrac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},i)|\sigma_x^2}\left(\tfrac{\hat{a}_{1h}}{\hat{b}_h}|\mathcal{S}(\mathcal{N},i)\cap\{h\}| + \tfrac{\hat{a}_{2h}}{\hat{b}_h}\textstyle\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k^{\mathcal{N}} + \tfrac{\hat{a}_{3h}}{\hat{b}_h}\tfrac{|\mathcal{S}(\mathcal{N},i)\cap\mathcal{S}(\mathcal{N},h)|\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N},h)|\sigma_x^2}\right)\sigma_\epsilon^2.\end{aligned} \tag{4.17}$$

Step 2: Recall that the following conditions hold for social clusters $\mathcal{C}_1$ and $\mathcal{C}_2$:

$$\begin{aligned}&\text{(i)} && \mathcal{C}_1 \text{ and } \mathcal{C}_2 \text{ are tight-knit,} \\ &\text{(ii)} && \mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset, \text{ and} \\ &\text{(iii)} && |\mathcal{C}_1| = |\mathcal{C}_2| > 1.\end{aligned} \tag{4.18}$$

In this step, we show that there exists $\Pi^{\mathcal{N}} \in \mathbb{R}$ such that $\pi_h^{\mathcal{N}} = \pi_{h'}^{\mathcal{N}} = \Pi^{\mathcal{N}}$ for all $h \in \mathcal{C}_1$ and $h' \in \mathcal{C}_2$ if σ_L^2 is sufficiently large. First, let us prove that there exists $\Pi_1^{\mathcal{N}} \in \mathbb{R}$ such that $\pi_h^{\mathcal{N}} = \Pi_1^{\mathcal{N}}$ for all $h \in \mathcal{C}_1$ when σ_L^2 is large enough. Suppose not. Then for any given level of σ_L^2, there exists some $r \in \mathcal{C}_1$ such that $\pi_r^{\mathcal{N}} \geq \pi_h^{\mathcal{N}}$ for all $h \in \mathcal{C}_1$ and $\pi_r^{\mathcal{N}} > \pi_s^{\mathcal{N}}$ for some $s \in \mathcal{C}_1$.

Following Lemma 1 and (4.18),

$$\begin{aligned}
\pi_r^{\mathcal{N}} &= \gamma^{\mathcal{N}} \left(\frac{\hat{a}_{1r}}{\hat{b}_r} + \sum_{m \in \{m' : \mathcal{S}(\mathcal{N}, m') \cap \mathcal{C}_1 \neq \emptyset\}} \frac{\hat{a}_{3m}}{\hat{b}_m} \frac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N}, m)| \sigma_x^2} \right) \\
&= \gamma^{\mathcal{N}} \left(\frac{\left(\sum_{j=1}^n \pi_j^{\mathcal{N}} - \sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}}\right)\left(\sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}} - |\mathcal{C}_1| \pi_r^{\mathcal{N}}\right)}{\rho\left((|\mathcal{C}_1|-1)\left(\sum_{j=1}^n (\pi_j^{\mathcal{N}})^2 |\sigma_\epsilon^2 + (\gamma^{\mathcal{N}})^2 \sigma_L^2\right) - \left(\sum_{k \in \mathcal{C}_1} \pi_k\right)^2 \sigma_\epsilon^2 + \pi_r \left(2 \sum_{k \in \mathcal{C}_1} \pi_k - |\mathcal{C}_1| \pi_r\right) \sigma_\epsilon^2\right)} \right. \\
&\qquad \left. + \sum_{m \in \{m' : \mathcal{S}(\mathcal{N}, m') \cap \mathcal{C}_1 \neq \emptyset\}} \frac{\hat{a}_{3m}}{\hat{b}_m} \frac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N}, m)| \sigma_x^2} \right).
\end{aligned}$$

Choose σ_L^2 large enough so that

$$\begin{aligned}
& \frac{\sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}} - |\mathcal{C}_1| \pi_r^{\mathcal{N}}}{(|\mathcal{C}_1| - 1)\left(\sum_{j=1}^n (\pi_j^{\mathcal{N}})^2 \sigma_\epsilon^2 + (\gamma^{\mathcal{N}})^2 \sigma_L^2\right) - \left(\sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}}\right)^2 \sigma_\epsilon^2 + \pi_r^{\mathcal{N}} \left(2 \sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}} - |\mathcal{C}_1| \pi_r^{\mathcal{N}}\right) \sigma_\epsilon^2} \\
< & \frac{\sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}} - |\mathcal{C}_1| \pi_s^{\mathcal{N}}}{(|\mathcal{C}_1| - 1)\left(\sum_{j=1}^n (\pi_j^{\mathcal{N}})^2 \sigma_\epsilon^2 + (\gamma^{\mathcal{N}})^2 \sigma_L^2\right) - \left(\sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}}\right)^2 \sigma_\epsilon^2 + \pi_s^{\mathcal{N}} \left(2 \sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}} - |\mathcal{C}_1| \pi_s^{\mathcal{N}}\right) \sigma_\epsilon^2}.
\end{aligned}$$

Then

$$\begin{aligned}
\pi_r^{\mathcal{N}} &< \gamma^{\mathcal{N}} \left(\frac{\left(\sum_{j=1}^n \pi_j^{\mathcal{N}} - \sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}}\right)\left(\sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}} - |\mathcal{C}_1| \pi_s^{\mathcal{N}}\right)}{\rho\left((|\mathcal{C}_1|-1)\left(\sum_{j=1}^n (\pi_j^{\mathcal{N}})^2 \sigma_\epsilon^2 + (\gamma^{\mathcal{N}})^2 \sigma_L^2\right) - \left(\sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}}\right)^2 \sigma_\epsilon^2 + \pi_s^{\mathcal{N}} \left(2 \sum_{k \in \mathcal{C}_1} \pi_k^{\mathcal{N}} - |\mathcal{C}_1| \pi_s^{\mathcal{N}}\right) \sigma_\epsilon^2\right)} \right. \\
&\qquad \left. + \sum_{m \in \{m' : \mathcal{S}(\mathcal{N}, m') \cap \mathcal{C}_1 \neq \emptyset\}} \frac{\hat{a}_{3m}}{\hat{b}_m} \frac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N}, m)| \sigma_x^2} \right) \\
&= \gamma^{\mathcal{N}} \left(\frac{\hat{a}_{1s}}{\hat{b}_s} + \sum_{m \in \{m' : \mathcal{S}(\mathcal{N}, m') \cap \mathcal{C}_1 \neq \emptyset\}} \frac{\hat{a}_{3m}}{\hat{b}_m} \frac{\sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N}, m)| \sigma_x^2} \right) \\
&= \pi_s^{\mathcal{N}}.
\end{aligned}$$

Since $\pi_r^{\mathcal{N}} > \pi_s^{\mathcal{N}}$, we have a clear contradiction. This proves that $\pi_h^{\mathcal{N}} = \Pi_1^{\mathcal{N}}$ for all $h \in \mathcal{C}_1$ when σ_L^2 is large enough.

Similarly, we can prove that there exists $\Pi_2^{\mathcal{N}} \in \mathbb{R}$ such that $\pi_{h'}^{\mathcal{N}} = \Pi_2^{\mathcal{N}}$ for all $h' \in \mathcal{C}_2$ if σ_L^2 is sufficiently large.

Now we only need to show that $\Pi_1^{\mathcal{N}} = \Pi_2^{\mathcal{N}} = \Pi^{\mathcal{N}}$ for some $\Pi^{\mathcal{N}} \in \mathbb{R}$. Suppose not. Then without loss of generality we can assume that $\Pi_1^{\mathcal{N}} > \Pi_2^{\mathcal{N}}$. Following Lemma 1 and (4.18),

$$\begin{aligned}
\Pi_1^{\mathcal{N}} &= \frac{\gamma^{\mathcal{N}}|\mathcal{C}_1|\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{C}_1|\sigma_x^2}\,\frac{(\sigma_\epsilon^2+\sigma_x^2)\left((\gamma^{\mathcal{N}})^2\sigma_L^2+\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2-2|\mathcal{C}_1|(\Pi_1^{\mathcal{N}})^2\sigma_\epsilon^2-\Pi_1^{\mathcal{N}}\sum_{j\notin\mathcal{C}_1\cup\mathcal{C}_2}\pi_j^{\mathcal{N}}\sigma_\epsilon^2\right)}{\rho\sigma_\epsilon^2\sigma_x^2\left((\gamma^{\mathcal{N}})^2\sigma_L^2+\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2-|\mathcal{C}_1|(\Pi_1^{\mathcal{N}})^2\sigma_\epsilon^2\right)}\\
&< \frac{\gamma^{\mathcal{N}}|\mathcal{C}_2|\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{C}_2|\sigma_x^2}\,\frac{(\sigma_\epsilon^2+\sigma_x^2)\left((\gamma^{\mathcal{N}})^2\sigma_L^2+\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2-2|\mathcal{C}_2|(\Pi_2^{\mathcal{N}})^2\sigma_\epsilon^2-\Pi_2^{\mathcal{N}}\sum_{j\notin\mathcal{C}_1\cup\mathcal{C}_2}\pi_j^{\mathcal{N}}\sigma_\epsilon^2\right)}{\rho\sigma_\epsilon^2\sigma_x^2\left((\gamma^{\mathcal{N}})^2\sigma_L^2+\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2-|\mathcal{C}_2|(\Pi_2^{\mathcal{N}})^2\sigma_\epsilon^2\right)}\\
&= \Pi_2^{\mathcal{N}}.
\end{aligned}$$

Thus, by contradiction, we must have $\Pi_1^{\mathcal{N}} = \Pi_2^{\mathcal{N}} = \Pi^{\mathcal{N}}$.

Step 3: Suppose that σ_L^2 is sufficiently large. Following Lemma 1 and Step 2, for all $h \in \mathcal{C}_1$ and $h' \in \mathcal{C}_2$,

$$\hat{a}_{1h} = \hat{a}_{1h'} = 0, \quad \hat{a}_{2h} = \hat{a}_{2h'} = \hat{a}_2, \quad \hat{a}_{3h} = \hat{a}_{3h'} = \hat{a}_3, \quad \hat{b}_h = \hat{b}_{h'} = \hat{b}$$

for some $\hat{a}_2$, $\hat{a}_3$, and $\hat{b}$. We can choose σ_L^2 large enough so that $\frac{\hat{a}_2}{\hat{b}} \neq 0$ and $\frac{\hat{a}_3}{\hat{b}} \neq 0$. From (4.16) and (4.17), if $i \in \mathcal{C}_1$, then

$$\begin{aligned}
&\mathrm{corr}(\tilde{z}_i^{\mathcal{N}}, \tilde{z}_h^{\mathcal{N}}) - \mathrm{corr}(\tilde{z}_i^{\mathcal{N}}, \tilde{z}_{h'}^{\mathcal{N}}) =\\
&= \frac{\left(\frac{a_3}{b}\right)^2\left(\frac{\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{C}_1|\sigma_x^2}\right)^2\sigma_\epsilon^2}{|\mathcal{C}_1|^2\left(2\Pi^{\mathcal{N}}\frac{\hat{a}_2}{\hat{b}}+\frac{\hat{a}_3}{\hat{b}}\frac{\sigma_x^2}{\sigma_x^2+|\mathcal{C}_1|\sigma_\epsilon^2}\right)^2\sigma_x^2+\left((|\mathcal{C}_1|-1)\left(\frac{\hat{a}_2}{\hat{b}}+\frac{\hat{a}_3}{\hat{b}}\frac{\sigma_x^2}{\sigma_x^2+|\mathcal{C}_1|\sigma_\epsilon^2}\right)^2+(n-|\mathcal{C}_1|)\left(\frac{\hat{a}_2}{\hat{b}}\Pi^{\mathcal{N}}\right)^2\right)\sigma_\epsilon^2+\left(\frac{\hat{a}_2}{\hat{b}}\gamma^{\mathcal{N}}\right)^2\sigma_L^2} > 0. \quad \square
\end{aligned}$$

Proof of Proposition 4: Let

$$\begin{aligned}
\tilde{p}^{\mathcal{N}_1} &= \pi_0^{\mathcal{N}_1} + \sum_{i=1}^n \pi_i^{\mathcal{N}_1}\,\tilde{\theta}_i - \gamma^{\mathcal{N}_1}\,\tilde{L},\\
\tilde{p}^{\mathcal{N}_2} &= \pi_0^{\mathcal{N}_2} + \sum_{i=1}^n \pi_i^{\mathcal{N}_2}\,\tilde{\theta}_i - \gamma^{\mathcal{N}_2}\,\tilde{L}.
\end{aligned}$$

Following Lemma 1, (3.2), also the fact that $\mathcal{S}(\mathcal{N}_1, i) \supsetneq \{i\}$ and $\mathcal{S}(\mathcal{N}_2, i) \supsetneq \{i\}$ $\forall i = 1, ..., n$, we have

$$
\begin{aligned}
\gamma^{\mathcal{N}} &= \left(1+\sum_{i=1}^{n}\left[\frac{(|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^{n}\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}}-\sum_{k\in\mathcal{S}(\mathcal{N},i)}\frac{\pi_k^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\right)\sigma_\epsilon^2\sigma_x^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^{n}(\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2\sigma_\epsilon^2-(\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2\sigma_\epsilon^2+\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\frac{\pi_k^{\mathcal{N}}}{\gamma^{\mathcal{N}}}-\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\right)^2\sigma_\epsilon^2\right)}\right]\right) \\
&\quad\times\left(-\sum_{i=1}^{n}\left[\frac{\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\frac{\pi_k^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\right)^2(\sigma_\epsilon^2+\sigma_x^2)\sigma_\epsilon^2-(|\mathcal{S}(\mathcal{N},i)|-1)(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_L^2-2\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\sum_{k\in\mathcal{S}(\mathcal{N},i)}\frac{\pi_k^{\mathcal{N}}}{\gamma^{\mathcal{N}}}(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^{n}(\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2\sigma_\epsilon^2-(\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2\sigma_\epsilon^2+\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\frac{\pi_k^{\mathcal{N}}}{\gamma^{\mathcal{N}}}-\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\right)^2\sigma_\epsilon^2\right)}\right.\right. \\
&\qquad+\frac{|\mathcal{S}(\mathcal{N},i)|(\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2-(|\mathcal{S}(\mathcal{N},i)|-1)\sum_{j=1}^{n}(\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_\epsilon^2}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^{n}(\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2\sigma_\epsilon^2-(\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2\sigma_\epsilon^2+\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\frac{\pi_k^{\mathcal{N}}}{\gamma^{\mathcal{N}}}-\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\right)^2\sigma_\epsilon^2\right)} \\
&\qquad\left.\left.-\frac{(|\mathcal{S}(\mathcal{N},i)|-1)\left(\left(\sum_{j=1}^{n}\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\right)^2(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2)\sigma_x^2-\sum_{j=1}^{n}\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\sigma_x^2\left(2\sum_{k\in\mathcal{S}(\mathcal{N},i)}\frac{\pi_k^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sum_{j=1}^{n}\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\sigma_x^2\right)\right)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^{n}(\frac{\pi_j^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2\sigma_\epsilon^2-(\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}})^2\sigma_\epsilon^2+\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\frac{\pi_k^{\mathcal{N}}}{\gamma^{\mathcal{N}}}-\frac{\pi_i^{\mathcal{N}}}{\gamma^{\mathcal{N}}}\right)^2\sigma_\epsilon^2\right)}\right]\right)^{-1}, \\
\pi_i^{\mathcal{N}} &= \gamma^{\mathcal{N}}\left(\frac{\sigma_\epsilon^2\sigma_x^2\left(\sum_{j=1}^{n}\pi_j^{\mathcal{N}}-\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k^{\mathcal{N}}\right)\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k^{\mathcal{N}}-|\mathcal{S}(\mathcal{N},i)|\pi_i^{\mathcal{N}}\right)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},i)|-1)\left(\sum_{j=1}^{n}(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2-(\pi_i^{\mathcal{N}})^2\sigma_\epsilon^2+(\gamma^{\mathcal{N}})^2\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k^{\mathcal{N}}-\pi_i^{\mathcal{N}}\right)^2\sigma_\epsilon^2\right)}+\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N},m)\cap\{i\}|\sigma_x^2\times\right. \\
&\quad\left.\frac{(|\mathcal{S}(\mathcal{N},m)|-1)\left(\sigma_L^2(\gamma^{\mathcal{N}})^2+\sigma_\epsilon^2\sum_{j=1}^{n}(\pi_j^{\mathcal{N}})^2\right)-|\mathcal{S}(\mathcal{N},m)|\sigma_\epsilon^2(\pi_m^{\mathcal{N}})^2-\sigma_\epsilon^2\sum_{k\in\mathcal{S}(\mathcal{N},m)}\pi_k^{\mathcal{N}}\sum_{j=1}^{n}\pi_j^{\mathcal{N}}+\sigma_\epsilon^2\pi_i^{\mathcal{N}}\left(\sum_{k\in\mathcal{S}(\mathcal{N},m)}\pi_k^{\mathcal{N}}+\sum_{j=1}^{n}\pi_j^{\mathcal{N}}\right)}{\rho\,\sigma_\epsilon^2\sigma_x^2\left((|\mathcal{S}(\mathcal{N},m)|-1)\left(\sum_{j=1}^{n}(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2-(\pi_m^{\mathcal{N}})^2\sigma_\epsilon^2+(\gamma^{\mathcal{N}})^2\sigma_L^2\right)-\left(\sum_{k\in\mathcal{S}(\mathcal{N},m)}\pi_k^{\mathcal{N}}-\pi_m^{\mathcal{N}}\right)^2\sigma_\epsilon^2\right)}\right).
\end{aligned}
$$

for $\mathcal{N}\in\{\mathcal{N}_1,\mathcal{N}_2\}$. Thus,

$$
\lim_{\sigma_L^2\to\infty}\gamma^{\mathcal{N}} = \frac{\rho\sigma_\epsilon^2\sigma_x^2}{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N},i)|\sigma_x^2}, \tag{4.19a}
$$

$$
\lim_{\sigma_L^2\to\infty}\pi_i^{\mathcal{N}} = \frac{\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N},m)\cap\{i\}|\,\sigma_x^2}{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N},i)|\sigma_x^2},\quad i=1,...,n,\ \ \mathcal{N}\in\{\mathcal{N}_1,\mathcal{N}_2\}. \tag{4.19b}
$$

Using (4.19a)-(4.19b), we obtain

$$
\begin{aligned}
\lim_{\sigma_L^2\to\infty}\frac{\mathrm{var}_L(\tilde{p}^{\mathcal{N}_1})}{\mathrm{var}_L(\tilde{p}^{\mathcal{N}_2})} &= \left(\frac{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N}_2,i)|\sigma_x^2}{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N}_1,i)|\sigma_x^2}\right)^2, \\
\lim_{\sigma_L^2\to\infty}\frac{\mathrm{var}_I(\tilde{p}^{\mathcal{N}_1})}{\mathrm{var}_I(\tilde{p}^{\mathcal{N}_2})} &= \left(\frac{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N}_2,i)|\sigma_x^2}{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N}_1,i)|\sigma_x^2}\right)^2\times \\
&\quad\frac{\left(\sum_{j=1}^{n}\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N}_1,m)\cap\{j\}|\right)^2\sigma_x^4+\sum_{j=1}^{n}(\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N}_1,m)\cap\{j\}|)^2\sigma_\epsilon^2\sigma_x^2}{\left(\sum_{j=1}^{n}\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N}_2,m)\cap\{j\}|\right)^2\sigma_x^4+\sum_{j=1}^{n}(\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N}_2,m)\cap\{j\}|)^2\sigma_\epsilon^2\sigma_x^2} \\
&= \left(\frac{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N}_2,i)|\sigma_x^2}{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N}_1,i)|\sigma_x^2}\right)^2\times \\
&\quad\frac{\left(\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N}_1,m)|\right)^2\sigma_x^4+\sum_{j=1}^{n}(\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N}_1,m)\cap\{j\}|)^2\sigma_\epsilon^2\sigma_x^2}{\left(\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N}_2,m)|\right)^2\sigma_x^4+\sum_{j=1}^{n}(\sum_{m=1}^{n}|\mathcal{S}(\mathcal{N}_2,m)\cap\{j\}|)^2\sigma_\epsilon^2\sigma_x^2}, \\
\lim_{\sigma_L^2\to\infty}\frac{\mathrm{var}(\tilde{p}^{\mathcal{N}_1})}{\mathrm{var}(\tilde{p}^{\mathcal{N}_2})} &= \left(\frac{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N}_2,i)|\sigma_x^2}{n\sigma_\epsilon^2+\sum_{i=1}^{n}|\mathcal{S}(\mathcal{N}_1,i)|\sigma_x^2}\right)^2.
\end{aligned}
$$

Now it is straightforward to see that for sufficiently large σ_L^2

(a) $\mathrm{var}_L(\tilde{p}^{\mathcal{N}_1}) < \mathrm{var}_L(\tilde{p}^{\mathcal{N}_2})$ if $\sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_1, i)\right| > \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_2, i)\right|$,

(b) $\mathrm{var}_I(\tilde{p}^{\mathcal{N}_1}) > \mathrm{var}_I(\tilde{p}^{\mathcal{N}_2})$ if $\sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_1, i)\right| = \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_2, i)\right|$ and $\sum_{i=1}^n \left(\sum_{m=1}^n \left|\mathcal{S}(\mathcal{N}_1, m) \cap \{i\}\right|\right)^2 > \sum_{i=1}^n \left(\sum_{m=1}^n \left|\mathcal{S}(\mathcal{N}_2, m) \cap \{i\}\right|\right)^2$,

(c) $\mathrm{var}(\tilde{p}^{\mathcal{N}_1}) < \mathrm{var}(\tilde{p}^{\mathcal{N}_2})$ if $\sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_1, i)\right| > \sum_{i=1}^n \left|\mathcal{S}(\mathcal{N}_2, i)\right|$. □

Proof of Remark 1: Using Lemma 1, it can be verified that $\mathcal{N}^*$ has unique linear equilibrium price and that it is of the form

$$\tilde{p}^{\mathcal{N}^*} = \pi_0^* + \pi_1^* \tilde{\theta}_1 + \pi_2^* \sum_{i=2}^n \tilde{\theta}_i + \gamma^* \tilde{L}, \tag{4.20}$$

where

$$\begin{aligned}
\pi_1^* &= \gamma^* \frac{n}{\rho\sigma_\epsilon{}^2 + \frac{(n-1)q^*\sigma_\epsilon{}^2}{(n-1)(q^*)^2\sigma_\epsilon{}^2 + \sigma_L{}^2} + (n-1)\frac{(n-2)q^*\sigma_\epsilon{}^2}{(n-2)(q^*)^2\sigma_\epsilon{}^2 + \sigma_L{}^2}}, \\
\pi_2^* &= \gamma^* q^*, \\
\gamma^* &= \frac{1 + \frac{1}{\rho}\frac{(n-1)q}{(n-1)(q^*)^2\sigma_\epsilon{}^2 + \sigma_L{}^2} + \frac{1}{\rho}\frac{(n-1)(n-2)q^*}{(n-2)(q^*)^2\sigma_\epsilon{}^2 + \sigma_L{}^2}}{\frac{n}{\rho\sigma_x{}^2} + \frac{2n-1}{\rho\sigma_\epsilon{}^2} + \frac{1}{\rho}\frac{(n-1)^2(q^*)^2}{(n-1)(q^*)^2\sigma_\epsilon{}^2 + \sigma_L{}^2} + \frac{1}{\rho}\frac{(n-1)(n-2)^2(q^*)^2}{(n-2)(q^*)^2\sigma_\epsilon{}^2 + \sigma_L{}^2}}, \\
q^* &= \sqrt[3]{\frac{\sigma_L{}^2}{2(n-2)\rho\sigma_\epsilon{}^4}} \left(\sqrt[3]{1 + \sqrt{1 + \frac{4}{27}\frac{\rho^2\sigma_L{}^2\sigma_\epsilon{}^2}{n-2}}} - \sqrt[3]{-1 + \sqrt{1 + \frac{4}{27}\frac{\rho^2\sigma_L{}^2\sigma_\epsilon{}^2}{n-2}}} \right)
\end{aligned}$$

Also following Lemma 1, $\mathcal{N}^o$ has a unique linear equilibrium price and it is of the form

$$\tilde{p}^{\mathcal{N}^o} = \pi_0^o + \pi^o \sum_{i=1}^n \tilde{\theta}_i + \gamma^o \tilde{L}, \tag{4.21}$$

where

$$\begin{aligned}
\pi^o &= \gamma^o q^o, \\
\gamma^o &= \frac{1 + \frac{1}{\rho}\frac{n(n-1)q^o}{((n-1)(q^o)^2\sigma_\epsilon{}^2 + \sigma_L{}^2)}}{n\,\frac{\sigma_x{}^2 + \sigma_\epsilon{}^2}{\rho\sigma_x{}^2\sigma_\epsilon{}^2} + \frac{1}{\rho}\frac{n(n-1)(q^o)^2}{((n-1)(q^o)^2\sigma_\epsilon{}^2 + \sigma_L{}^2)}}, \\
q^o &= \sqrt[3]{\frac{\sigma_L{}^2}{2(n-1)\rho\sigma_\epsilon{}^4}} \left(\sqrt[3]{1 + \sqrt{1 + \frac{4}{27}\frac{\rho^2\sigma_L{}^2\sigma_\epsilon{}^2}{n-1}}} - \sqrt[3]{-1 + \sqrt{1 + \frac{4}{27}\frac{\rho^2\sigma_L{}^2\sigma_\epsilon{}^2}{n-1}}} \right).
\end{aligned}$$

Given $\sigma_L^2 = s\, n^2$, equations (4.20) and (4.21) yield

$$\lim_{n\to\infty} \mathrm{var}(\tilde{p}^{\mathcal{N}^*}) = \left(\frac{\sigma_x^2}{\sigma_x^2+\sigma_\epsilon^2}\right)^2 (\sigma_x^2+\sigma_\epsilon^2) + \left(\frac{\rho\sigma_x^2\sigma_\epsilon^2}{\sigma_x^2+\sigma_\epsilon^2}\right)^2 s, \tag{4.22a}$$

$$\lim_{n\to\infty} \mathrm{var}(\tilde{p}^{\mathcal{N}^o}) = \left(\frac{\rho\sigma_x^2\sigma_\epsilon^2}{\sigma_x^2+\sigma_\epsilon^2}\right)^2 s, \tag{4.22b}$$

respectively. The inequalities (3.4) and (3.5) in Remark 1 follow immediately from (4.22a)-(4.22b). $\square$

Proof of Proposition 5: Let $\{\tilde{p}^{\mathcal{N}} : \mathcal{N} \in \Omega\}$ be the set of linear equilibrium prices satisfying

$$\tilde{p}^{\mathcal{N}} = \pi_0^{\mathcal{N}} + \sum_{i=1}^n \pi_i^{\mathcal{N}}\, \tilde{\theta}_i - \gamma^{\mathcal{N}}\, \tilde{L}$$

with $0 < \lim_{\sigma_L^2\to\infty} \gamma^{\mathcal{N}} < \infty,\ 0 < \lim_{\sigma_L^2\to\infty} \pi_i^{\mathcal{N}} < \infty,\ i = 1, ..., n,$ for all $\mathcal{N} \in \Omega$. Following Lemma 1 and equations (4.7b), (4.10b) in its proof, for any $\mathcal{N} \in \Omega$ and $i \in \{1, ..., n\}$

$$\mathrm{var}\left(\tilde{X}\middle|\tilde{\theta}_i, \mathrm{E}[\tilde{X}|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}], \tilde{p}^{\mathcal{N}}\right) = \frac{\sigma_\epsilon^2\sigma_x^2\left(\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2-(\pi_i^{\mathcal{N}})^2\sigma_\epsilon^2+(\gamma^{\mathcal{N}})^2\sigma_L^2\right)}{(\sigma_\epsilon^2+\sigma_x^2)\left(\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2+\left(\sum_{j=1}^n\pi_j^{\mathcal{N}}\right)^2\sigma_x^2+(\gamma^{\mathcal{N}})^2\sigma_L^2\right)-(\pi_i^{\mathcal{N}})^2\sigma_\epsilon^4-2\pi_i^{\mathcal{N}}\sum_{j=1}^n\pi_j^{\mathcal{N}}\sigma_\epsilon^2\sigma_x^2} \tag{4.23a}$$

if $\mathcal{S}(\mathcal{N}, i) = \{i\}$ and

$$\begin{aligned}\mathrm{var}\left(\tilde{X}\middle|\tilde{\theta}_i, \mathrm{E}[\tilde{X}|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N},i)}], \tilde{p}^{\mathcal{N}}\right) = {} & \sigma_\epsilon^2\sigma_x^2\Big((|\mathcal{S}(\mathcal{N},i)|-1)\left(\textstyle\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2-(\pi_i^{\mathcal{N}})^2\sigma_\epsilon^2+(\gamma^{\mathcal{N}})^2\sigma_L^2\right)-\left(\textstyle\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k^{\mathcal{N}}-\pi_i^{\mathcal{N}}\right)^2\sigma_\epsilon^2\Big)\times \\ & \Big[(|\mathcal{S}(\mathcal{N},i)|-1)\left(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2\right)\left(\textstyle\sum_{j=1}^n(\pi_j^{\mathcal{N}})^2\sigma_\epsilon^2+\left(\sum_{j=1}^n\pi_j^{\mathcal{N}}\right)^2\sigma_x^2+(\gamma^{\mathcal{N}})^2\sigma_L^2\right) \\ & +2\pi_i^{\mathcal{N}}\textstyle\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k^{\mathcal{N}}\left(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2\right)\sigma_\epsilon^2-|\mathcal{S}(\mathcal{N},i)|(\pi_i^{\mathcal{N}})^2\left(\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sigma_x^2\right)\sigma_\epsilon^2 \\ & +(|\mathcal{S}(\mathcal{N},i)|-1)\textstyle\sum_{j=1}^n\pi_j^{\mathcal{N}}\sigma_x^2\left(2\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k^{\mathcal{N}}\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N},i)|\sum_{j=1}^n\pi_j\sigma_x^2\right) \\ & -\left(\textstyle\sum_{k\in\mathcal{S}(\mathcal{N},i)}\pi_k^{\mathcal{N}}\right)^2(\sigma_\epsilon^2+\sigma_x^2)\sigma_\epsilon^2\Big]^{-1}.\end{aligned} \tag{4.23b}$$

if $\mathcal{S}(\mathcal{N}, i) \supsetneq \{i\}$. Now note that $|\mathcal{S}(\mathcal{N}, i)| = 1$ if $\mathcal{S}(\mathcal{N}, i) = \{i\}$. From equations (4.23a)-(4.23b), we obtain that for all $\mathcal{N}_1, \mathcal{N}_2 \in \Omega$ and $i \in \{1, ..., n\}$

$$\begin{aligned}\lim_{\sigma_L^2\to\infty} & \left(\mathrm{var}\left(\tilde{X}\middle|\tilde{\theta}_i, \mathrm{E}[\tilde{X}|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N}_1,i)}], \tilde{p}^{\mathcal{N}_1}\right) - \mathrm{var}\left(\tilde{X}\middle|\tilde{\theta}_i, \mathrm{E}[\tilde{X}|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N}_2,i)}], \tilde{p}^{\mathcal{N}_2}\right)\right) \\ & = \frac{\sigma_\epsilon^2\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N}_1,i)|\sigma_x^2} - \frac{\sigma_\epsilon^2\sigma_x^2}{\sigma_\epsilon^2+|\mathcal{S}(\mathcal{N}_2,i)|\sigma_x^2}.\end{aligned} \tag{4.24}$$

Thus, if $|\mathcal{S}(\mathcal{N}_1, i)| > |\mathcal{S}(\mathcal{N}_2, i)|$, then

$$\mathrm{var}\left(\tilde{X}\middle|\tilde{\theta}_i, \mathrm{E}[\tilde{X}|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N}_1,i)}], \tilde{p}^{\mathcal{N}_1}\right) < \mathrm{var}\left(\tilde{X}\middle|\tilde{\theta}_i, \mathrm{E}[\tilde{X}|\{\tilde{\theta}_k\}_{k\in\mathcal{S}(\mathcal{N}_2,i)}], \tilde{p}^{\mathcal{N}_2}\right)$$

for sufficiently large σ_L^2. This proves part (a) of the proposition.

Let $\hat{\mathcal{N}}$ be a social network satisfying $\mathcal{S}(\hat{\mathcal{N}}, i) = \{1, ..., n\}$ for all $i \in \{1, ..., n\}$. Choose an arbitrary social network $\mathcal{N} \in \Omega$ satisfying $\mathcal{S}(\mathcal{N}, h) \subsetneq \{1, ..., n\}$ for some $h \in \{1, ..., n\}$. Then

$$\frac{\sigma_\epsilon^2 \sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\mathcal{N}, h)| \sigma_x^2} \;>\; \frac{\sigma_\epsilon^2 \sigma_x^2}{\sigma_\epsilon^2 + n\, \sigma_x^2} \;=\; \frac{\sigma_\epsilon^2 \sigma_x^2}{\sigma_\epsilon^2 + |\mathcal{S}(\hat{\mathcal{N}}, h)| \sigma_x^2}.$$

Following (4.24), this implies

$$\mathrm{var}\left(\tilde{X} \middle| \tilde{\theta}_h, \mathrm{E}[\tilde{X} | \{\tilde{\theta}_k\}_{k \in \mathcal{S}(\mathcal{N}, h)}], \tilde{p}^{\mathcal{N}}\right) > \mathrm{var}\left(\tilde{X} \middle| \tilde{\theta}_h, \mathrm{E}[\tilde{X} | \{\tilde{\theta}_k\}_{k \in \mathcal{S}(\hat{\mathcal{N}}, h)}], \tilde{p}^{\hat{\mathcal{N}}}\right)$$

for sufficiently large σ_L^2. So, for sufficiently large σ_L^2, no social network $\mathcal{N}$ can informationally dominate $\hat{\mathcal{N}}$ for the set of equilibrium prices $\{\tilde{p}^{\hat{\mathcal{N}}}, \tilde{p}^{\mathcal{N}}\}$. This proves part (b) of the proposition.
$\square$

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