Flexible contracts

P. Ghirardato, P. Gottardi, J.-M. Tallon

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1 Introduction

An objective of this work is the study of the consequences of the presence of unforeseen contingencies or other forms of higher order uncertainty in the agents' decision process for the properties of contractual arrangements. In particular, the presence of unforeseen contingencies or private information over the states of the world are often advocated (e.g. Maskin (2002), p. 726) as one of the main sources of the incompleteness of contracts that is observed. However, the attempts to show this formally have not so far been very successful. We contend that this is, to some extent, to attribute to the fact that, while there has been considerable progress in the study of the consequences of the presence of such forms of uncertainty for the individual decision making process, the same is not true for their consequences in a multiperson framework. In that case, the degree of information of any agent relative to the others also needs to be modelled.

To illustrate this, note that, following Kreps (1992) and the more recent work of Dekel, Lipman, Rustichini (2001), the effects of the realization of unforeseen contingencies for an individual's decision can be modelled as the effect of the realization of some shock affecting the agent's preferences. However, in a multiperson contracting framework one has also to specify whether such shocks are or not observable to other parties, or equivalently whether unforeseen contingencies give rise to some form of informational asymmetry. The common assumption in the literature is that, even though states may not be described, or foreseen, the realization of their consequences are observed by all contractual parties (though possibly not verifiable by outside parties). In such situation, as shown by Maskin and Tirole (1999) appropriate incentive mechanisms can be used to extract agents' information at no cost.

The line of work undertaken in this paper departs from the existing literature on this point. We consider situations where such individual shocks are only privately observable by the party which is affected by them, or more generally where unforeseen contingencies give rise to informational asymmetries among the parties. As a consequence these contingencies have a clear effect on what can be achieved through contracts, since additional incentive constraints have now to be faced to deal with this new source of asymmetric information.

Furthermore if, in this situation, the parties of the contract have multiple though common priors this has an interesting interplay with their private information: the contract design may in fact induce the use of different beliefs by the parties of the contract and hence affect the advantages of trade among them (on this, see also some recent work by Eliaz and Spiegler (2005)).

In this paper we intend to investigate whether the presence of unforeseen contingencies may provide a rationale for the use of flexible, or discretionary, contracts instead of fixed or rigid contracts. We consider a simple contracting situation between a principal and an agent. The agent's choice of his action is not observable by the principal but we suppose that, at the time of contracting, the principal has the ability to predefine the set of actions, or possible tasks, that will be available to the agent when he has to make his choice: thus the principal can choose to specify some fixed tasks for the agent or alternatively leave him some discretion. In particular, the principal can impose very tight restrictions on the set of actions available to the agent and leave him no discretion, thus overcoming any agency problem caused by the agent's unobservable action choice. However, in the presence of unforeseen contingencies, affecting for instance the agent's cost – or the effectiveness - of undertaking certain tasks, it may be preferable to leave the agent discretion in his choice, by letting the set of actions available to him be sufficiently large, so that the agent's choice can be adjusted on the basis of the realization of the different contingencies. At the same time, granting flexibility also has some costs, since giving discretion, when the agent's actual choice of action, as well as the contingencies under which it is made, are only privately observed, creates an agency problem; the larger the set of possible actions available to the agent the more severe the incentive problem faced by the principal. The design of the agent's compensation in a flexible contract will then have to be more complex, as the use of appropriate high-powered incentives will be needed, and some agency costs incurred. The issue is important as such trade-off naturally arises when the architecture of organizations is evaluated.

In such a simple set-up we can determine the optimal flexible contract and compare it to the optimal rigid (or non discretionary) contracts. We can then investigate how various factors (in particular, the agents' attitude towards risk, and the severity of the contingencies that are not foreseen, ...) affect the choice between flexible and fixed contracts. We will also look at the effects of allowing for the presence of multiple priors when principal and agent evaluate the consequences of future unforeseen events. In this case the design of the flexible contract may be such as to induce the same or different beliefs between the agent and the principal. Can differences in beliefs enhance the benefits of flexible contracts? And can they be such as to more than compensate the costs of the asymmetric information among the parties?

2 The Set-up

We consider a contractual relationship between a principal, say a firm, and an agent, say a worker. The worker has two possible actions, x and y. The output generated by each action is uncertain: it can be either high ($\bar{R}$) or low ($\underline{R}$). The cost of undertaking action $z = x, y$ is c_z.

In addition, the probability of the high outcome with action x (resp. y) is also uncertain and depends on some event $\theta \in \{\theta_1, \theta_2\}$: it is $\pi(x, \theta)$ (resp. $\pi(y, \theta)$).

We assume the contract is written before the realization of any source of uncertainty (i.e., before the output and θ are realized). In addition, the realization of the output is publicly observable while the effort is only privately known by the agent exerting it. Furthermore θ, describing some unforeseen contingency, or event affecting the execution/profitability of the different possible tasks/actions, is only privately observed by the agent, not by the principal (or any third party). On the other hand, the probability of occurrence of θ_1 is assumed to be known and equal to p.

Although the action undertaken by the agent is not observable, we assume that the principal can a priori impose some restrictions over the set of actions available to the agent. To understand the nature of this restriction we can think, for instance, at a situation where the principal can leave the agent free to choose among different types software (in which case both x and y are available to the agent) or can decide to install only one software on the agent's computer (in which case only one action is available to the agent). In this framework, therefore a compensation contract is a specification of a set of admissible actions $A \subseteq \{x, y\}$ together with a wage payment w from the principal to the agent, where w can depend on the realized level of the output and the agent's announcement about the realization of the event θ. Let $\bar{w}_i$ (resp. $\underline{w}_i$) denote the compensation paid to the agent when the output is $\bar{R}$ (resp. $\underline{R}$) and the (declared) state is $\theta_i, i = 1, 2$.

In particular, we would like to distinguish the case where the full menu of possible actions is available to the agent, $A = \{x, y\}$ from the cases where, respectively, only action x (y) is available to the agent. We refer to the contract in the first case as a *flexible contract*, since the agent has the flexibility and the discretion to choose the action he feels as more appropriate to him (and suitable incentives should be specified in the contract to induce the agent to make a choice also in the principal's interest). In the second case we say on the other hand the contract is *rigid*, and can then be of type x or of type y according to which action is prescribed to the agent.

It may help to think of the following time-line:

$t = 0$ The contract is signed, specifying the payments due to the agent for each possible realization of the output and each announcement of the agent regarding θ. In addition the contract specifies the set $A \subseteq \{x, y\}$ of possible actions available to the agent.

$t = 1$ θ is observed by the agent. Agent announces a value for θ.

$t = 2$ The agent undertakes an action $z \in A$, not observable by the principal.

$t = 3$ Output is revealed (i.e., uncertainty about output is resolved and output is observed)

$t = 4$ Compensation is paid to the agent, according to the realized output level and announcement of the agent.

Observe that at the time in which the contract is signed there is symmetric information among the parties, the agent does not know the realization of the unforeseen contingency. Asymmetric information will arise at a later stage, when the agent learns some information about the profitability of the different actions, and decides then which action to take.

Remark 1 *We ignore here the possibility of renegotiation, in particular at the time in which the realization of θ is learnt by the agent* ($t = 1$).

Our main goal is to investigate in this set-up the relative profitability of flexible and rigid contracts. While the flexible contract offers the agent the opportunity to choose the best action in each possible contingency, the delegation of the choice to the agent, since the action is not observable, creates an agency problem, and hence the wage schedule will have to satisfy a set of appropriate incentive compatibility constraints. On the other hand, in the rigid contract since the agent has no discretion, there is no agency problem, but the action implemented cannot be adjusted to the different contingencies.

We focus our attention on the case where the values of these parameters satisfy the following condition:

Assumption 1

- $c_x > c_y$, *i.e.,* $\Delta c \equiv c_x - c_y > 0$.
- $\pi(x, \theta_1) > \pi(x, \theta_2) > \pi(y, \theta_2) > \pi(y, \theta_1)$ *[the spread in the probability of success is higher in state θ_1 than in state θ_2]*

Thus action x corresponds to a higher (more costly) level of effort. Moreover, while action x is always more productive than action y, its additional productivity, relative to action y, is uncertain: in state θ_1 action x is more productive, relatively to action y than in state θ_2.

The principal is the residual claimant of the output and is risk neutral. His payoff, when action z_i, $i = 1, 2$, is implemented in state θ_i, is then given by the expected profit:

$$p[\pi(z_1, \theta_1)(\bar{R} - \bar{w}_1)) + (1 - \pi(z_1, \theta_1))(\underline{R} - \underline{w}_1)] \\ +(1 - p)[\pi(z_2, \theta_2)(\bar{R} - \bar{w}_2) + (1 - \pi(z_2, \theta_2))(\underline{R} - \underline{w}_2)]$$

The agent is risk averse and has the following preferences

Assumption 2 *The agent has the (non separable) utility function over consumption/compensation and effort:* $u(w, z) = -\frac{e^{-a(w-z)}}{a}$, *with* $a > 0$.

It is then convenient to normalize the agent's reservation utility as $-\frac{e^{-a\bar{u}}}{a}$.

Remark 2 *The consideration of a utility function that is nonseparable in effort and wages allows us to study the comparative statics properties of the optimal contract with respect to the agent's level of risk aversion -one of our objectives-, as in this case changes in the curvature of the agent's utility function captures changes in the agent's attitude towards risk in the compensation (Moreover, in the case of CARA preferences, risk attitude can be captured by the single parameter a). On the other hand, with a utility function separable in effort and wages, as $u(w) - v(e)$, it would be hard to interpret the concavity of the utility function over the wages $u(.)$ as reflecting risk aversion (as it also reflects the rate of substitution between effort and consumption/wages).*

3 Optimal flexible contract

3.1 Derivation of the optimal flexible contract

Under the restriction imposed by Assumption 1 on the parameter values it is clear that the optimal action profile to be implemented at a flexible contract is action x in state θ_1 (where action x is relatively more productive) and action y in θ_2. Hence the optimal flexible contract implementing these actions is obtained as solution of the following programme:

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]$$

$$\begin{cases} \pi(x,\theta_1)u(\bar{w}_1-c_x)+(1-\pi(x,\theta_1))u(\underline{w}_1-c_x)\geq \pi(x,\theta_1)u(\bar{w}_2-c_x)+(1-\pi(x,\theta_1))u(\underline{w}_2-c_x) \\ \pi(x,\theta_1)u(\bar{w}_1-c_x)+(1-\pi(x,\theta_1))u(\underline{w}_1-c_x)\geq \pi(y,\theta_1)u(\bar{w}_2-c_y)+(1-\pi(y,\theta_1))u(\underline{w}_2-c_y) \\ \pi(x,\theta_1)u(\bar{w}_1-c_x)+(1-\pi(x,\theta_1))u(\underline{w}_1-c_x)\geq \pi(y,\theta_1)u(\bar{w}_1-c_y)+(1-\pi(y,\theta_1))u(\underline{w}_1-c_y) \\ \\ \pi(y,\theta_2)u(\bar{w}_2-c_y)+(1-\pi(y,\theta_2))u(\underline{w}_2-c_y)\geq \pi(x,\theta_2)u(\bar{w}_1-c_x)+(1-\pi(x,\theta_2))u(\underline{w}_1-c_x) \\ \pi(y,\theta_2)u(\bar{w}_2-c_y)+(1-\pi(y,\theta_2))u(\underline{w}_2-c_y)\geq \pi(x,\theta_2)u(\bar{w}_2-c_x)+(1-\pi(x,\theta_2))u(\underline{w}_2-c_x) \\ \pi(y,\theta_2)u(\bar{w}_2-c_y)+(1-\pi(y,\theta_2))u(\underline{w}_2-c_y)\geq \pi(y,\theta_2)u(\bar{w}_1-c_y)+(1-\pi(y,\theta_2))u(\underline{w}_1-c_y) \\ \\ p[\pi(x,\theta_1)u(\bar{w}_1-c_x)+(1-\pi(x,\theta_1))u(\underline{w}_1-c_x)]+ \\ \qquad (1-p)[\pi(y,\theta_2)u(\bar{w}_2-c_y)+(1-\pi(y,\theta_2))u(\underline{w}_2-c_y)]\geq \bar{u} \end{cases}$$

The above formulation of the programme is for the case of general, nonseparable preferences over wage payments and effort. the case. When, in additions preferences are described by CARA utility functions, as in Assumption 2, we can rewrite the program as follows:

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]$$

$$\begin{cases} (IC1) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)} \\ (IC2) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \\ (IC3) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} \\ (IC4) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)} \\ (IC5) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)} \\ (IC6) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)} \\ \\ (PC) & p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}]+ \\ & \quad (1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}] \leq e^{-a\bar{u}} \end{cases} \tag{P}$$

We will show that at an optimal solution of this problem, only the constraints (IC2), (IC6) and (PC) are binding and derive, at the same time, some properties of the optimal compensation. Will do this in several steps. All these steps assume that there exists an optimal solution; we will provide in Proposition 2 below some conditions ensuring a solution exists..

Step 1: At an optimal solution $\bar{w}_2 \geq \underline{w}_2$.

Proof.

Suppose not, that is, $\bar{w}_2 < \underline{w}_2$.

Then, it is immediate to show, given that $c_y < c_x$, $\pi(y,\theta_1) < \pi(x,\theta_1)$, and $\pi(y,\theta_2) < \pi(x,\theta_2)$, that both (IC1) and (IC5) are slack. Start with (IC1): the right hand side of (IC1) is strictly greater than the right hand side of (IC2) and hence, (IC1) is slack. For (IC5), rewrite the constraint as:

$$[\pi(y,\theta_2)e^{-a\bar{w}_2}+(1-\pi(y,\theta_2))e^{-a\underline{w}_2}]e^{ac_y} \leq [\pi(x,\theta_2)e^{-a\bar{w}_2}+(1-\pi(x,\theta_2))e^{-a\underline{w}_2}]e^{ac_x}$$

Then, under the assumption, the expression in bracket in the left hand side is strictly smaller than the one in the right hand side, which implies, together with the order on the cost, that (IC5) is slack.

We now show that if $\bar{w}_2 < \underline{w}_2$, then it is possible to find an improvement for the Principal by pushing $\bar{w}_2$ and $\underline{w}_2$ closer. Consider $\Delta\bar{w}_2 > 0$ and $\Delta\underline{w}_2 < 0$ (i.e. a discrete change in $\bar{w}_2, \underline{w}_2$) such that:

(i) $\pi(y,\theta_2)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_y)} = \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}$

and,

(ii) $\pi(y,\theta_2)\Delta\bar{w}_2+(1-\pi(y,\theta_2))\Delta\underline{w}_2 < 0$

Note that it is possible to find such a $\Delta\bar{w}_2$ and $\Delta\underline{w}_2$ by concavity of the utility function. By condition (ii), we can conclude that this change improves the Principal's profit. It remains to show that it is feasible and satisfies the remaining incentive and the participation constraints.

(IC3) is trivially satisfied since it does not depend on $\Delta\bar{w}_2$ and $\Delta\underline{w}_2$. (IC4) and (IC6) are satisfied by construction, given condition (i) and the same is true for (PC). Thus, it remains to show that (IC2) holds. Given that the left hand side of (IC2) remains unchanged, it is enough to show that:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_y)}$$

This follows from condition (i) and the fact that $\pi(y,\theta_1) < \pi(y,\theta_2)$. Indeed, (i) is equivalent to $\pi(y,\theta_2)[e^{-a(\bar{w}_2+\Delta\bar{w}_2)} - e^{-a\bar{w}_2}] + (1-\pi(y,\theta_2))[e^{-a(\underline{w}_2+\Delta\underline{w}_2)} - e^{-a\underline{w}_2}] = 0$. The first term is negative while the second is positive, so we have, given that $\pi(y,\theta_1) < \pi(y,\theta_2)$,

$\pi(y,\theta_1)[e^{-a(\bar{w}_2+\Delta\bar{w}_2)} - e^{-a\bar{w}_2}] + (1-\pi(y,\theta_1))[e^{-a(\underline{w}_2+\Delta\underline{w}_2)} - e^{-a\underline{w}_2}] > 0$, which yields the desired result. □

Step 2: At an optimal solution $\bar{w}_1 > \underline{w}_1$.
Proof. This is a direct consequence of (IC3). □

Step 3: At an optimal solution (IC2) binds.
Proof. We distinguish two cases, according to whether $\underline{w}_2 = \bar{w}_2$ or $\underline{w}_2 < \bar{w}_2$.

Case 1.: $\underline{w}_2 = \bar{w}_2 \equiv w_2$.

In that event, (IC5) is automatically satisfied and therefore can be dropped. Furthermore, (IC2) implies (IC1) which thus can also be dropped. Now, by Step 2 $\underline{w}_1 < \bar{w}_1$. Hence, given that $\pi(y,\theta_2) > \pi(y,\theta_1)$, it is possible to show that (IC2) and (IC6) imply (IC3), which can therefore be dropped. Obviously, (IC2) and (IC4) can not be simultaneously binding. We now establish that (IC2) has to bind and therefore (IC4) is slack.

Assume not, i.e., (IC2) is slack and consider (an infinitesimal change) $d\bar{w}_1 < 0$, $d\underline{w}_1 = 0$ and $dw_2 > 0$. Since (IC2) is slack, for sufficiently small such quantities it continues to hold. (IC4) and (IC6) remain satisfied. Choosing $d\bar{w}_1 = -\frac{(1-p)e^{-a(w_2-c_y)}}{p\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}}dw_2$ ensures that the participation constraint continues to hold. By construction, the change in the objective function is equal to $(1-p)\left[\frac{e^{-a(w_2-c_y)}}{e^{-a(\bar{w}_1-c_x)}} - 1\right]dw_2$. Given that $dw_2 > 0$, this quantity is positive (hence leading to an increase in the objective function) if $e^{-a(w_2-c_y)} > e^{-a(\bar{w}_1-c_x)}$, that is, if $\bar{w}_1 > w_2 + \Delta c$; we show next that this property always holds in the case under consideration ($\underline{w}_2 = \bar{w}_2$). In such case (IC2) can in fact be rewritten as follows:

$$\pi(x,\theta_1)e^{-a\bar{w}_1} + (1-\pi(x,\theta_1))e^{-a\underline{w}_1} \leq e^{-a(w_2+\Delta c)},$$

which in turn implies, together with the property $\bar{w}_1 > \underline{w}_1$ established in Step 2, that $e^{-a\bar{w}_1} < e^{-a(w_2+\Delta c)}$, and therefore $\bar{w}_1 > w_2 + \Delta c$.

Hence, whenever (IC2) is slack we can find a perturbation of the wage bill that increases the Principal's profit, contradicting optimality of the contract. Therefore (IC2) has to bind (and hence (IC4) is slack).

Case 2.: $\underline{w}_2 < \bar{w}_2$.

Assume (IC2) is slack and consider (a discrete change) $\Delta\underline{w}_2 > 0$ and $\Delta\bar{w}_2 < 0$ such that: (i) $\pi(y,\theta_2)\Delta\bar{w}_2 + (1-\pi(y,\theta_2)\Delta\underline{w}_2 < 0$ and (ii) $\pi(y,\theta_2)e^{-a(\bar{w}_2+\Delta\bar{w}_2)} + (1-\pi(y,\theta_2))e^{-a(\bar{w}_2+\Delta\underline{w}_2)} = \pi(y,\theta_2)e^{-a\bar{w}_2} + (1-\pi(y,\theta_2))e^{-a\bar{w}_2}$. Such numbers exist by strict concavity of u.

Notice that (IC3), (IC4), (IC6) and (PC) are unaffected by these changes and thus continue to hold. We now check (IC1). The left hand side is unchanged and we therefore need to show that: $\pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_x)}$, which is equivalent to

$$\pi(x,\theta_1)[e^{-a\bar{w}_2} - e^{-a(\bar{w}_2+\Delta\bar{w}_2)}] + (1-\pi(x,\theta_1))[e^{-a\underline{w}_2} - e^{-a(\underline{w}_2+\Delta\underline{w}_2)}] \leq 0$$

But this holds as a consequence of (ii), given that $\Delta\underline{w}_2 > 0$ and $\Delta\bar{w}_2 < 0$ and $\pi(x,\theta_1) > \pi(y,\theta_2)$. Thus, (IC1) continues to hold.

It remains to check (IC5). By construction, the left hand side is unaffected by the change. Given that $\pi(x,\theta_2) > \pi(y,\theta_2)$, one can replicate the argument showing that (IC1) holds to prove that (IC5) holds as well. □

Step 4: At an optimal solution (IC4) is slack.

Proof. Given that $\bar{w}_2 \geq \underline{w}_2$ and $\pi(y,\theta_2) \geq \pi(y,\theta_1)$, we have $\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}$. From the previous step, we know (IC2) is binding, and hence

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}$$

Given that $\bar{w}_1 > \underline{w}_1$ and $\pi(x,\theta_1) \geq \pi(x,\theta_2)$, this establishes that (IC4) is slack, i.e.

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} < \pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)}$$

□

Step 5: At an optimal solution (IC5) is slack.

Proof. If $\bar{w}_2 = \underline{w}_2$, this is obvious. Consider next the case $\bar{w}_2 > \underline{w}_2$. Then, $\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}$. From step 3 we know that (IC2) binds, i.e., $\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} = \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}$.

Now, by (IC1), $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)}$ and hence, since $\bar{w}_2 > \underline{w}_2$ and $\pi(x,\theta_1) > \pi(x,\theta_2)$, $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} < \pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)}$. As a consequence,

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} < \pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)}$$

showing that (IC5) is slack. □

Step 6: At an optimal solution, $\bar{w}_1 \geq \bar{w}_2$ and $\underline{w}_1 \leq \underline{w}_2$. Furthermore, if $\underline{w}_1 = \underline{w}_2$, then it must be the case that $\bar{w}_1 = \bar{w}_2$.

Proof. Rewrite (IC1) and (IC6) as follows:

$$\pi(x,\theta_1)\left[e^{-a\bar{w}_1} - e^{-a\bar{w}_2}\right] \leq (1-\pi(x,\theta_1))\left[e^{-a\underline{w}_2} - e^{-a\underline{w}_1}\right] \quad (1)$$

$$\pi(y,\theta_2)\left[e^{-a\bar{w}_2} - e^{-a\bar{w}_1}\right] \leq (1-\pi(y,\theta_2))\left[e^{-a\underline{w}_1} - e^{-a\underline{w}_2}\right] \quad (2)$$

Assume $\bar{w}_1 < \bar{w}_2$, then (1) implies that $\underline{w}_1 > \underline{w}_2$ and (1) and (2) yield that:

$$\frac{\pi(x,\theta_1)}{1-\pi(x,\theta_1)} \leq \frac{e^{-a\underline{w}_2} - e^{-a\underline{w}_1}}{e^{-a\bar{w}_1} - e^{-a\bar{w}_2}} \leq \frac{\pi(y,\theta_2)}{1-\pi(y,\theta_2)}$$

But this is not possible given that $\pi(y,\theta_2) < \pi(x,\theta_1)$. Hence, $\bar{w}_1 \geq \bar{w}_2$. A similar argument establishes that $\underline{w}_1 \leq \underline{w}_2$.

Finally, suppose that $\underline{w}_1 = \underline{w}_2$. Then, using the fact that (IC2) is binding, one can rewrite (IC3) as follows:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}$$

which yields $\bar{w}_1 = \bar{w}_2$, since we assumed that $\underline{w}_1 = \underline{w}_2$ and we proved above that $\bar{w}_1 \geq \bar{w}_2$. . □

Step 7: At an optimal solution (IC3) is slack if $\underline{w}_1 < \underline{w}_2$. If $\underline{w}_1 = \underline{w}_2$, (IC3) is automatically satisfied as equality.

Proof. Use (IC2), which is binding, to rewrite (IC3) as follows:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} \quad (3)$$

If $\underline{w}_1 < \underline{w}_2$, (3) is equivalent, given that $\bar{w}_1 \geq \bar{w}_2$, to

$$\frac{e^{-a\bar{w}_2} - e^{-a\bar{w}_1}}{e^{-a\underline{w}_1} - e^{-a\underline{w}_2}} \leq \frac{1-\pi(y,\theta_1)}{\pi(y,\theta_1)}$$

But we know by (IC6) that

$$\frac{e^{-a\bar{w}_2} - e^{-a\bar{w}_1}}{e^{-a\underline{w}_1} - e^{-a\underline{w}_2}} \leq \frac{1-\pi(y,\theta_2)}{\pi(y,\theta_2)}$$

and hence, since $\pi(y,\theta_1) < \pi(y,\theta_2)$, (IC3) is slack.

If $\underline{w}_1 = \underline{w}_2$, then we know that $\bar{w}_1 = \bar{w}_2$ and (3) - hence (IC3) - is automatically satisfied. □

Step 8: At an optimal solution (IC1) and (IC6) cannot be simultaneously binding if $\underline{w}_1 < \underline{w}_2$. If $\underline{w}_1 = \underline{w}_2$, then they are both automatically satisfied (as equalities).

Proof. Assume $\underline{w}_1 < \underline{w}_2$ and observe that if (IC2) and (IC6) were binding, one would have

$$\frac{1-\pi(x,\theta_1)}{\pi(x,\theta_1)} = \frac{e^{-a\bar{w}_2} - e^{-a\bar{w}_1}}{e^{-a\underline{w}_1} - e^{-a\underline{w}_2}} = \frac{1-\pi(y,\theta_2)}{\pi(y,\theta_2)}$$

a contradiction. □

Step 9: At an optimal solution, if $\underline{w}_1 < \underline{w}_2$, (IC6) binds.

Proof. Assume $\underline{w}_1 < \underline{w}_2$ and (IC6) is slack and consider changing $\bar{w}_1$ and $\underline{w}_1$ by respectively $\Delta\bar{w}_1 < 0$ and $\Delta\underline{w}_1 > 0$ such that, (i) $\pi(x,\theta_1)\Delta\bar{w}_1 + (1-\pi(x,\theta_1))\Delta\underline{w}_1 < 0$ and (ii), $\pi(x,\theta_1)e^{-a(\bar{w}_1+\Delta\bar{w}_1)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1+\Delta\underline{w}_1)} = \pi(x,\theta_1)e^{-a\bar{w}_1} + (1-\pi(x,\theta_1))e^{-a\underline{w}_1}$. Such a change exists by strict concavity of the utility function and provides higher profit to the principal.

Furthermore, this change does not affect (IC1), (IC2), and (PC) and is feasible given that (IC3), (IC4), (IC5) and (IC6) are slack. Hence, (IC6) has to be binding at an optimal solution whenever $\underline{w}_1 < \underline{w}_2$. □

The results established so far can be summarized in the following:

Proposition 1 *At an optimal flexible contract the compensation exhibits the following properties:* $\bar{w}_1 \geq \bar{w}_2 \geq \underline{w}_2 \geq \underline{w}_1$, *and* $\bar{w}_1 > \underline{w}_1$. *Furthermore:*

(i) if $\underline{w}_2 > \underline{w}_1$, *then* $\bar{w}_1 > \bar{w}_2$ *and (IC2) and (IC6) are binding, while (IC1), (IC3), (IC4) and (IC5) are slack.*

(ii) if $\underline{w}_2 = \underline{w}_1$, *then* $\bar{w}_1 = \bar{w}_2$ *and (IC2) binds, while (IC1), (IC3), and (IC6) are automatically satisfied ((IC1) and (IC6) as equalities), and (IC4) and (IC5) are slack.*

Remark 3 *Everything done so far depends only on the fact that the utility function can be decomposed as* $u(w-c) = u(w)u(-c)$ *with* u *(strictly) concave and increasing. Thus the specific CARA form is not necessary for the above results.*

A corollary to Proposition 1 is that the optimal flexible contract can be obtained as a solution to the simpler programme below:

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] + (1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]$$

$$\begin{cases} (IC2) \;\; \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} = \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \\ (IC6) \;\; \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} = \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)} \\ (PC) \;\; p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}] + \\ \qquad (1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}] \leq e^{-a\bar{u}} \\ (W) \;\; \bar{w}_1 \geq \bar{w}_2 \\ (W') \;\; \bar{w}_2 \geq \underline{w}_2 \end{cases} \qquad (\tilde{P})$$

Observe there is no need to add the constraint $\underline{w}_2 \geq \underline{w}_1$ since it is implied by (W) and (IC6).

Proposition 2 *A necessary and sufficient condition for the existence of a solution to problem* $(\tilde{P})$ *(and hence also to* (P)*) is that* $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$.

Proof. (*Necessity*) Assume that (IC2) and (IC6) hold as equalities and wages are ordered $\bar{w}_1 \geq \bar{w}_2 \geq \underline{w}_2 \geq \underline{w}_1$.

From (IC6) holding as equality, we get that $\pi(y,\theta_2)(e^{-a\bar{w}_2} - e^{-a\bar{w}_1}) = (1-\pi(y,\theta_2))(e^{-a\underline{w}_1} - e^{-a\underline{w}_2})$ and therefore, given that the two terms are non-negative and $\pi(y,\theta_2) > \pi(y,\theta_1)$:

$$\pi(y,\theta_1)(e^{-a\bar{w}_2} - e^{-a\bar{w}_1}) \leq (1-\pi(y,\theta_1))(e^{-a\underline{w}_1} - e^{-a\underline{w}_2})$$

i.e., $\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}$.

Using (IC2) this implies that

$$\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}$$

or

$$(\pi(x,\theta_1)e^{a\Delta c} - \pi(y,\theta_1))e^{-a(\bar{w}_1-c_y)} \leq ((1-\pi(y,\theta_1)) - (1-\pi(x,\theta_1)e^{a\Delta c}))e^{-a(\underline{w}_1-c_y)}$$

A necessary condition for the above inequality to hold is that $((1-\pi(y,\theta_1))-(1-\pi(x,\theta_1)e^{a\Delta c})) \geq 0$, i.e., $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$.

(*Sufficiency*)

The two binding constraints (IC2) and (IC6) enable one to solve for $\bar{z}_1 = e^{-a\bar{w}_1}$ and $\underline{z}_1 \equiv e^{-a\underline{w}_1}$ as a function of $\bar{z}_2 \equiv e^{-a\bar{w}_2}$ and $\underline{z}_2 \equiv e^{-a\underline{w}_2}$.

We get:

$$\begin{aligned}
\bar{z}_1 &= \frac{\big((1-\pi(y,\theta_2))[\pi(y,\theta_1)\bar{z}_2 + (1-\pi(y,\theta_1))\underline{z}_2]e^{-a\Delta c} - (1-\pi(x,\theta_1))[\pi(y,\theta_2)\bar{z}_2 + (1-\pi(y,\theta_2))\underline{z}_2]\big)}{\pi(x,\theta_1) - \pi(y,\theta_2)} \\
\underline{z}_1 &= \frac{\big(\pi(x\theta_1)[\pi(y,\theta_2)\bar{z}_2 + (1-\pi(y,\theta_2))\underline{z}_2] - \pi(y,\theta_2)[\pi(y,\theta_1)\bar{z}_2 + (1-\pi(y,\theta_1))\underline{z}_2]e^{-a\Delta c}\big)}{\pi(x,\theta_1) - \pi(y,\theta_2)}
\end{aligned}$$

We now want to establish that under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, it is possible to find $0 \leq \bar{z}_2 \leq \underline{z}_2$ such that:

$$\begin{aligned}
\bar{z}_1 &> 0 \\
\bar{z}_1 &\leq \bar{z}_2 \\
\underline{z}_2 &\leq \underline{z}_1 \\
\bar{z}_2 &\leq \underline{z}_2
\end{aligned}$$

The first inequality is equivalent, under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, to

$$\frac{(1-\pi(x,\theta_1))\pi(y,\theta_2) - (1-\pi(y,\theta_2))\pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_2))[(1-\pi(y,\theta_1))e^{-a\Delta c} - (1-\pi(x,\theta_1))]} < \frac{\underline{z}_2}{\bar{z}_2} \tag{4}$$

The next two inequalities are actually equivalent (again under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$) to the same inequality:

$$\frac{\pi(x,\theta_1) - \pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_1))e^{-a\Delta c} - (1-\pi(x,\theta_1))} \geq \frac{\underline{z}_2}{\bar{z}_2} \tag{5}$$

Thus, to show that we can find some $\bar{z}_2 \leq \underline{z}_2$ such that (4) and (5) hold, we need to establish that, under the assumption that $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, the following holds:

$$\max\left(1, \frac{(1-\pi(x,\theta_1))\pi(y,\theta_2) - (1-\pi(y,\theta_2))\pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_2))[(1-\pi(y,\theta_1))e^{-a\Delta c} - (1-\pi(x,\theta_1))]}\right) < \frac{\pi(x,\theta_1) - \pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_1))e^{-a\Delta c} - (1-\pi(x,\theta_1))}$$

Straightforward computation shows that this is indeed the case.

Hence, $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$ is a necessary and sufficient condition for a solution to (IC2) and (IC6) as equality to exist, that satisfy the required ordering. ■

Before solving problem $(\tilde{P})$, observe that one can rewrite it, by changing variables, as a problem with a (strictly) concave objective and linear constraints. Specifically, let $z = e^{-aw}$, then, with obvious notation, solving $(\tilde{P})$ is equivalent to solving the following problem:

$$\max_{\bar{z}_1,\underline{z}_1,\bar{z}_2,\underline{z}_2} \quad p[\pi(x,\theta_1)(\bar{R} + \tfrac{\log \bar{z}_1}{a})) + (1-\pi(x,\theta_1))(\underline{R} + \tfrac{\log \underline{z}_1}{a})] \\ +(1-p)[\pi(y,\theta_2)(\bar{R} + \tfrac{\log \bar{z}_2}{a}) + (1-\pi(y,\theta_2))(\underline{R} + \tfrac{\log \underline{z}_2}{a})]$$

$$\left\{\begin{array}{l} (IC2) \;\; \pi(x,\theta_1)e^{ac_x}\bar{z}_1 + (1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1 = \pi(y,\theta_1)e^{ac_y}\bar{z}_2 + (1-\pi(y,\theta_1))e^{ac_y}\underline{z}_2 \\ (IC6) \;\; \pi(y,\theta_2)e^{ac_y}\bar{z}_2 + (1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2 = \pi(y,\theta_2)e^{ac_y}\bar{z}_1 + (1-\pi(y,\theta_2))e^{ac_y}\underline{z}_1 \\ \\ (PC) \;\; p[\pi(x,\theta_1)e^{ac_x}\bar{z}_1 + (1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1] + \\ \qquad\qquad (1-p)[\pi(y,\theta_2)e^{ac_y}\bar{z}_2 + (1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2] \leq e^{-a\bar{u}} \\ \\ (W) \;\; \bar{z}_1 \leq \bar{z}_2 \\ (W') \;\; \bar{z}_2 \leq \underline{z}_2 \end{array}\right. \tag{$\tilde{P}^T$}$$

Proposition 3 *At a solution to the program $(P^{red,T})$, (PC) binds. Furthermore, we have that $\bar{w}_2 > \underline{w}_2$.*

Proof. Consider the program $(P^{red,T})$. Let λ_2, λ_6, λ_{PC}, λ_W, and $\lambda_{W'}$ denote the Lagrange multipliers associated to the constraints of this problem. The first order conditions obtained by differentiating the Lagrangean with respect to $\bar{z}_1, \bar{z}_2, \underline{z}_2, \underline{z}_1$ are then:

$$\begin{cases} (i) & \frac{p\pi(x,\theta_1)}{a\bar{z}_1} = \lambda_2\pi(x,\theta_1)e^{ac_x} - \lambda_6\pi(y,\theta_2)e^{ac_y} + \lambda_{PC}p\pi(x,\theta_1)e^{ac_x} + \lambda_W \\ (ii) & \frac{p(1-\pi(x,\theta_1))}{a\underline{z}_1} = \lambda_2(1-\pi(x,\theta_1))e^{ac_x} - \lambda_6(1-\pi(y,\theta_2))e^{ac_y} \\ & \quad +\lambda_{PC}p(1-\pi(x,\theta_1))e^{ac_x} \\ (iii) & \frac{(1-p)\pi(y,\theta_2)}{a\bar{z}_2} = -\lambda_2\pi(y,\theta_1)e^{ac_y} + \lambda_6\pi(y,\theta_2)e^{ac_y} \\ & \quad +\lambda_{PC}(1-p)\pi(y,\theta_2)e^{ac_y} - \lambda_W + \lambda_{W'} \\ (iv) & \frac{(1-p)(1-\pi(y,\theta_2))}{a\underline{z}_2} = -\lambda_2(1-\pi(y,\theta_1))e^{ac_y} + \lambda_6(1-\pi(y,\theta_2))e^{ac_y} \\ & \quad +\lambda_{PC}(1-p)(1-\pi(y,\theta_2))e^{ac_y} - \lambda_{W'} \end{cases}$$

Multiplying each equation by the appropriate z, adding the four equations of the above system and using the fact that (IC2) and (IC6), in the above specification of the optimization problem, are written as equalities, yields the following:

$$\begin{aligned} \tfrac{1}{a} = {} & \lambda_{PC}[p\pi(x,\theta_1)e^{ac_x}\bar{z}_1 + p(1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1 \\ & +(1-p)\pi(y,\theta_2)e^{ac_y}\bar{z}_2 + (1-p)(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2] + \lambda_W[\bar{z}_1 - \bar{z}_2] + \lambda_{W'}[\bar{z}_2 - \underline{z}_2] \end{aligned}$$

Using the complementarity slackness condition, we get that $\lambda_W[\bar{z}_1 - \bar{z}_2] = \lambda_{W'}[\bar{z}_2 - \underline{z}_2] = 0$. Hence $\lambda_{PC} > 0$, which establishes that (PC) binds. Hence, we can conclude from the expression above that $\lambda_{PC} = \frac{e^{a\bar{u}}}{a}$.

Next we want to show that $\bar{w}_2 > \underline{w}_2$ or equivalently $\underline{z}_2 > \bar{z}_2$. Assume to the contrary that $\bar{z}_2 = \underline{z}_2 \equiv z_2$. We know in that case that (W) is slack (otherwise by (IC6) all wages would have to be equal, but this would contradict the fact that (IC2) binds) and hence $\lambda_W = 0$. Rewrite now the FOC (iii) and (iv) as:

$$\begin{cases} (iii) & \frac{(1-p)}{az_2} = -\lambda_2\frac{\pi(y,\theta_1)}{\pi(y,\theta_2)}e^{ac_y} + \lambda_6 e^{ac_y} + \lambda_{PC}(1-p)e^{ac_y} + \frac{\lambda_{W'}}{\pi(y,\theta_2)} \\ (iv) & \frac{(1-p)}{az_2} = -\lambda_2\frac{1-\pi(y,\theta_1)}{1-\pi(y,\theta_2)}e^{ac_y} + \lambda_6 e^{ac_y} + \lambda_{PC}(1-p)e^{ac_y} - \frac{\lambda_{W'}}{1-\pi(y,\theta_2)} \end{cases}$$

Thus, this implies that

$$-\lambda_2\frac{\pi(y,\theta_1)}{\pi(y,\theta_2)}e^{ac_y} + \frac{\lambda_{W'}}{\pi(y,\theta_2)} = -\lambda_2\frac{1-\pi(y,\theta_1)}{1-\pi(y,\theta_2)}e^{ac_y} - \frac{\lambda_{W'}}{1-\pi(y,\theta_2)}$$

or, after some simplification,

$$\lambda_{W'} = (\pi(y,\theta_1) - \pi(y,\theta_2))\lambda_2 e^{ac_y}$$

Given that $(\pi(y,\theta_1) - \pi(y,\theta_2)) < 0$ and that Lagrange multipliers have to be non negative, this implies that $\lambda_{W'} = \lambda_2 = 0$.

Next observe that (PC) as an equality together with (IC2) imply, if $\bar{z}_2 = \underline{z}_2 \equiv z_2$, that $z_2 = e^{-a(c_y+\bar{u})}$. Plug now the values of λ_{PC} and z_2 into equation (iii). Given that $\lambda_W = \lambda_{W'} =$

$\lambda_2 = 0$, we obtain that $\frac{(1-p)}{ae^{-a(c_y+\bar{u})}} = \lambda_6 + (1-p)\frac{e^{a(c_y+\bar{u})}}{a}$. But this implies that $\lambda_6 = 0$. Plugging this in FOC (i) and (ii) yields that $\bar{z}_1 = \underline{z}_1$ and hence by (IC6) that all wages are equal, which is as we noticed a contradiction. Hence we conclude that $\bar{z}_2 < \underline{z}_2$. ■

Using the constraints which we know are binding (IC2), (IC6) and (PC), it may be convenient, to find the solutions of problem $(P^{red,T})$, to solve such constraints so as to express all wages in terms of $\Delta z_2 \equiv \bar{z}_2 - \underline{z}_2$ (and, as shown above, $\Delta z_2 < 0$.)

$$\begin{cases} \bar{z}_1 = \frac{1}{\pi(x,\theta_1)-\pi(y,\theta_2)} \left\{ \left[(1-\pi(y,\theta_2))e^{-a\Delta c} - (1-\pi(x,\theta_1))\right] e^{-a(\bar{u}+c_y)} + \right. \\ \quad \left. (\pi(y,\theta_1)-\pi(y,\theta_2)) \left[(1-p)(1-\pi(y,\theta_2))e^{-a\Delta c} + p(1-\pi(x,\theta_1))\right] \Delta z_2 \right\} \\ \underline{z}_1 = \frac{1}{\pi(x,\theta_1)-\pi(y,\theta_2)} \left\{ \left[\pi(x,\theta_1) - \pi(y,\theta_2)e^{-a\Delta c}\right] e^{-a(\bar{u}+c_y)} + \right. \\ \quad \left. (\pi(y,\theta_2)-\pi(y,\theta_1)) \left[(1-p)\pi(y,\theta_2)e^{-a\Delta c} + p\pi(x,\theta_1)\right] \Delta z_2 \right\} \\ \bar{z}_2 = e^{-a(\bar{u}+c_y)} + (1-p\pi(y,\theta_1)-(1-p)\pi(y,\theta_2))\Delta z_2 \\ \underline{z}_2 = e^{-a(\bar{u}+c_y)} - (p\pi(y,\theta_1)+(1-p)\pi(y,\theta_2))\Delta z_2 \end{cases} \tag{6}$$

An important implication of the previous result is that

Corollary 1 *At the optimal flexible contract, the agent's utility in state θ_2 is strictly greater than his utility in state θ_1.*

Proof. This follows from the fact that (IC2) is binding together with the fact that $\pi(y,\theta_1) < \pi(y,\theta_2)$ and, as we have just shown, $\bar{w}_2 \geq \underline{w}_2$. ■

Combining the results above we obtain the following characterization of the optimal flexible contract:

Proposition 4 *Under Assumption 1, a necessary and sufficient condition for the existence of the optimal flexible contract is that $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, and the optimal flexible contract is the solution of the following simplified problem:*

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] + (1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]$$

$$\begin{cases} (IC2) \quad \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} = \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \\ (IC6) \quad \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} = \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)} \\ (PC) \quad p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}] + \\ \qquad (1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}] = e^{-a\bar{u}} \\ (W) \quad \bar{w}_1 \geq \bar{w}_2 \end{cases} \tag{P^{red}}$$

and exhibits the following properties:

$$\bar{w}_1 \geq \bar{w}_2 > \underline{w}_2 \geq \underline{w}_1.$$

It is interesting to point out that the optimal contract is characterized by a variable wage in state θ_2 even though in that state the low effort is implemented in that state. At the same time, as we argued, the utility of the compensation paid to the manager is higher in state θ_2 than in θ_1. The variability in w_2 and the lack of smoothing in the agent's utility can be justified as a way to reduce the variability in the compensation of the wage paid in θ_1: it can in fact be verified (see also the following sections) that (IC2), (IC6) and (PC) can all be satisfied as equality also with a constant level of w_2 - and hence with the sane utility for the agent in state θ_2 as in θ_1 - but this is suboptimal.

Remark 4 *If one were to extend the analysis to the case where the principal and the agent's prior belief over the realization of state θ_1 were different (respectively p^P and p^A), then most of the analysis above would still be valid, with two main exceptions: the first part of Step 3 of the proof of Proposition 1 and the proof above of the claim of Proposition 3 that at the optimal contract w_2 is not constant. We will argue in the next Sections that the proof of Step 3 can be extended to the case of different priors. Consider then the proof of Proposition 3: the argument in the last paragraph of the proof where it is claimed that $\lambda_6 = 0$ is no longer valid if $p^P < p^A$. We would have in fact $\frac{(1-p^P)}{ae^{-a(c_y+\bar{u})}} = \lambda_6 + (1-p^A)\frac{e^{a(c_y+\bar{u})}}{a}$, where p^P (resp. p^A) is the Principal's (resp. the Agent's) belief (probability of state θ_1). Hence, if $p^P > p^A$ the above equality implies $\lambda_6 < 0$, again a contradiction which allows us to establish the claim $\bar{w}_2 > \underline{w}_2$. On the other hand, if $p^P < p^A$ we no longer get a contradiction and thus are unable to establish the claim in that case. Hence in the case of different priors between principal and agent such that $p^P < p^A$ (i.e. the principal believes state θ_1 is less likely than what the agent believes, the optimal flexible contract may be characterized by a constant wage in state θ_2 : $\bar{w}_2 = \underline{w}_2$ and hence by full insurance between the θ_1 and θ_2 states. Note that when w_2 is constant the agent's utility is the same in the two θ states, while when $\bar{w}_2 > \underline{w}_2$ it is higher in state θ_2; it is then not surprising that the first situation may prove superior when the agent attributed a lower utility to state θ_2 than the principal.*

3.2 Optimal flexible contract when future events are commonly observable

An issue of interest is the comparison of the optimal flexible contract we derived with the optimal contract (still implementing actions x and y in states θ_1 and θ_2 respectively) when the realization of θ is observable. The latter is obtained as solution of the following problem:

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]$$

$$\begin{cases} (IC) \;\; \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} \\ \\ (PC) \;\; p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}]+ \\ \qquad (1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}] \leq e^{-a\bar{u}} \end{cases} \qquad (P^{SB})$$

The comparison of the two contracts allows then to capture the effects of the private information over the realization of θ. We have:

Proposition 5 *At an optimal contract (still implementing actions x and y) when θ is commonly observable we have $\bar{w}_2 = \underline{w}_2$ as well as $\bar{w}_1 > \underline{w}_1$. Moreover, the agent's utility is the same in state θ_1 as in θ_2.*

This shows that the variability of w_2 we found in the optimal flexible contract in Proposition 3 is due to the need to address the incentives concerning the agent's private information over θ (a lower variability in w_2 can only be achieved, as we already argued, at the cost of a higher variability of w_1).

Proof. Observe first that (IC) implies that $\bar{w}_1 > \underline{w}_1$. The property $\bar{w}_2 = \underline{w}_2$ can then be easily verified.

Consider then the first order conditions of problem (P^{SB}) with constant w_2:

$$\begin{cases} (i) \quad -p\pi(x,\theta_1)+\lambda_{IC}(-a\pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+a\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)})+ \\ \qquad \lambda_{PC}ap\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}=0 \\ \\ (ii) \quad -p(1-\pi(x,\theta_1))+\lambda_{IC}(-a(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}+a(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}+ \\ \qquad \lambda_{PC}ap(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}=0 \\ \\ (iii) \quad -(1-p)+(1-p)\lambda_{PC}ae^{-a(w_2-c_y)}=0 \end{cases}$$

Condition (iii) implies that $\lambda_{PC} = \frac{e^{a(w_2-c_y)}}{a} > 0$ and hence that (PC) is binding. Take now the summation of (i) and (ii) and use the complementary slackness condition (that says that $\lambda_{IC} \times (IC) = 0$), to obtain:

$$-p + a\lambda_{PC}p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}] = 0$$

Using the fact that (PC) is binding, this amounts to:

$$-p + a\lambda_{PC}[e^{-a\bar{u}}-(1-p)e^{-a(w_2-c_y)}] = 0$$

and finally, $e^{-a(w_2-c_y)} = e^{-a\bar{u}}$. Using again the fact that (PC) binds, we obtain that

$$\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} = e^{-a\bar{u}} = e^{-a(w_2-c_y)},$$

thus establishing the fact that at a solution of the above problem the utility of the agent is the same in state θ_1 and θ_2, in contrast with the property established in the previous section of the optimal flexible contract when θ is only privately observed by the agent. ■

4 The Choice between flexible and rigid contracts

We will analyze here the trade-off between the choice of flexible and rigid contracts. To this end we will compare the profits of the principal at the optimal flexible contract, which as shown in the previous section are obtained by substituting the values of the agent's compensation solving the problem in Proposition 4 into the Principal's objective function, with the profits of the optimal rigid contract. The optimal rigid contract implementing action z, $z = x, y$, in every state is obtained as solution of the following programme (note that the only constraint is given by PC, no incentive compatibility constraint appears here as the agent has no discretion over the choice of his action):

$$\begin{array}{ll} \max_{\bar{w}_1, \underline{w}_1, \bar{w}_2, \underline{w}_2} & p[\pi(z,\theta_1)(\bar{R}-\bar{w}_1) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ & \quad +(1-p)[\pi(z,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(z,\theta_2))(\underline{R}-\underline{w}_2)] \end{array}$$

$$\begin{array}{ll} (PC) & p[\pi(z,\theta_1)e^{-a(\bar{w}_1-c_z)} + (1-\pi(z,\theta_1))e^{-a(\underline{w}_1-c_z)}] + \\ & \quad (1-p)[\pi(z,\theta_2)e^{-a(\bar{w}_2-c_z)} + (1-\pi(z,\theta_2))e^{-a(\underline{w}_2-c_z)}] = e^{-a\bar{u}} \end{array} \quad (P^{rig})$$

Its solution is very simple in the present framework: the wage should be constant ($\bar{w}_1 = \underline{w}_1 = \bar{w}_2 = \underline{w}_2 = w_z$) and such as to equal the expected cost of undertaking action z (i.e., simply determined at the level set by the participation constraint). In particular:

i) Fixed x contract: the compensation is $w_x = \bar{u} + c_x$, while profits are:

$$[p\pi(x,\theta_1) + (1-p)\pi(x,\theta_2)]\bar{R} + [p(1-\pi(x,\theta_1)) + (1-p)(1-\pi(x,\theta_2))]\underline{R} - \bar{u} - c_x$$

ii) Fixed y contract: the compensation is $w_y = \bar{u} + c_y$, and profits:

$$[p\pi(y,\theta_1) + (1-p)\pi(y,\theta_2)]\bar{R} + [p(1-\pi(y,\theta_1)) + (1-p)(1-\pi(y,\theta_2))]\underline{R} - \bar{u} - c_y$$

We are now ready to compare the profitability of the optimal flexible contracts with that of the optimal rigid contracts and study then the effects of the various parameters (the cost of undertaking the different types of actions, the probabilities, describing both the relative productivity of the different actions as well as the relevance of the uncertainty affecting them, preferences and in particular the agent's risk attitude) on the optimality of one versus the other type of contract.

As we said, in the optimal flexible contract the agent's action can be better tailored to the different circumstances under which the agent may find himself to operate, however there is an agency cost in delegating the choice of the action to the agent since the action is not observable and the agent's objectives are not aligned to those of the principal. We should expect therefore that the advantages of flexibility will be higher the bigger is the difference between the productivity of the two types of actions in state θ_1 relative to the other state θ_2 as well as the smaller is the 'agency cost' which has to be paid to implement the action profile x, y.

4.1 Comparative statics with respect to the degree of uncertainty and the actions' cost

Our findings for the comparative statics properties with respect to the degree of uncertainty faces by principal and agent (as measured by the probability levels π) and the cost of undertaking the different types of actions c appear in line with the above intuition:

The profitability of the flexible contract, relative to the rigid contracts, increases if:

i) $\pi(x, \theta_1)$ (profitability of high effort in state 1) increases, or $\pi(y, \theta_1)$ decreases (high effort in state θ_1 is more productive relative to low effort - as well as relative to state θ_2; note that this should also have the effect of decreasing the agency costs of implementing action x in state θ_1);

ii) $\pi(x, \theta_2)$ decreases, or $\pi(y, \theta_2)$ increases (the difference between the productivity of high and low effort in state θ_2 is decreased, symmetric to the previous case);

iii) c_x decreases (the cost of exerting high effort relative to low effort decreases; this makes high effort relative more productive than low effort - in both states - and should also reduce the agency cost of implementing high effort in the flexible contract)**;**

iv) the cost of effort c_y decreases.

The above properties are derived from the analysis of a series of numerical examples, where they proved to be robust. We conjecture they have a general validity.

Consider next the case of discrete changes of the probabilities $\pi(y, \theta_1), \pi(y, \theta_2)$, leading from a situation where $\pi(x, \theta_1) > \pi(x, \theta_2) > \pi(y, \theta_2) > \pi(y, \theta_1)$, as in the case considered so far (under Assumption 1), to one where $\pi(x, \theta_1) > \pi(x, \theta_2) > \pi(y, \theta_1) > \pi(y, \theta_2)$. We can show that, while in the first situation there is an additional agency cost attached to the agent's private information over the state θ, this is no longer true in the latter situation:

Claim 1 *Assume $\pi(x, \theta_1) > \pi(x, \theta_2) > \pi(y, \theta_1) > \pi(y, \theta_2)$. Then the optimal flexible contract is the same as the optimal contract obtained when θ is observable.*

Proof. Notice first that the characterization of the optimal flexible contract for the case where θ is observable provided in Proposition 5 remains valid also when $\pi(x,\theta_1) > \pi(x,\theta_2) > \pi(y,\theta_1) > \pi(y,\theta_2)$. This can be easily verified by checking that the assumption $\pi(y,\theta_1) > \pi(y,\theta_2)$ does not play any role in the derivation of the result in section 3.2.

As proved in Proposition 5 the optimal contract when θ is observable satisfies the following conditions:

(a) $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} = e^{-a\bar{u}}$

(b) $e^{-a(w_2-c_y)} = e^{-a\bar{u}}$

(c) $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}$

We show next that any contract that satisfies the three conditions above is feasible when θ is not observable. We check the 6 Incentive Compatibility and Participation constraints in turn.

(IC1) amounts to $e^{-a\bar{u}} \leq e^{-a\bar{u}}e^{a\Delta c}$, which is satisfied given that $\Delta c > 0$.

(IC2) amounts to $e^{-a\bar{u}} \leq e^{-a\bar{u}}$.

(IC3) holds by condition (c). Note it implies that $\bar{w}_1 > \underline{w}_1$.

(IC4) amounts to $e^{-a\bar{u}} = \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} < \pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)}$, which is satisfied given that $\pi(x,\theta_1) > \pi(x,\theta_2)$ and $\bar{w}_1 > \underline{w}_1$.

(IC5) is satisfied given that w_2 is constant and $\Delta c > 0$.

(IC6) amounts to $e^{-a\bar{u}} \leq \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}$. By condition (a), this is equivalent to $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}$. Given that $\pi(y,\theta_1) > \pi(y,\theta_2)$ and $\bar{w}_1 > \underline{w}_1$, we have also $\pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} < \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}$, and hence condition (c) implies that $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}$ which is what we wanted to establish.

(PC) holds by conditions (a) and (b).

We have thus shown that, when $\pi(x,\theta_1) > \pi(x,\theta_2) > \pi(y,\theta_1) > \pi(y,\theta_2)$, the optimal contract for the case where θ is observable is a feasible contract also when θ is unobservable. Hence it is the optimal contract also in that case. ■

The above result shows that a discrete change of the probabilities, given by a decrease in $\pi(y,\theta_2)$ and/or an increase in $\pi(y,\theta_1)$ such that the new values satisfy the condition $\pi(y,\theta_1) > \pi(y,\theta_2)$, implies that the agency costs of the flexible contract will be lower.

4.2 Comparative statics with respect to risk aversion

Another important determinant of the agency costs of the flexible contract is given by the agent's risk attitude. As shown above, the compensation paid at the rigid contracts is independent of the agent's risk attitude (as described, in the case of CARA preferences, by the single parameter a); this on the other hand, matters for the optimal flexible contract.

To see the consequences of the agent's risk attitude for the agency costs of the flexible contract, we consider first the extreme case where the agent is also risk neutral. In that case, agency costs are zero as the first best can be implemented, i.e. the principal can attain the same level of profits as he would get when all incentive compatibility constraints are ignored.

Claim 2 *When the agent is risk neutral the optimal flexible contract is first best optimal. The profit level is* $p[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1))\underline{R}]+(1-p)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2))\underline{R}]-\bar{u}-pc_x-(1-p)c_y$ *and the optimal compensation is given by*

$$\begin{aligned}
\bar{w}_1 &= \bar{u}+c_x+\frac{1-\pi(x,\theta_1)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\Delta c \\
\underline{w}_1 &= \bar{u}+c_x-\frac{\pi(x,\theta_1)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\Delta c \\
\bar{w}_2 &= \underline{w}_2=\bar{u}+c_y
\end{aligned} \tag{7}$$

Proof. Under risk neutrality, programme (P) yielding the optimal flexible contract implementing actions x in θ_1 and y in θ_2 becomes:

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad \begin{aligned} & p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ & +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \end{aligned}$$

$$\left\{\begin{array}{l}
\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1 \geq \pi(x,\theta_1)\bar{w}_2+(1-\pi(x,\theta_1))\underline{w}_2 \\
\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-c_x \geq \pi(y,\theta_1)\bar{w}_2+(1-\pi(y,\theta_1))\underline{w}_2-c_y \\
\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-c_x \geq \pi(y,\theta_1)\bar{w}_1+(1-\pi(y,\theta_1))\underline{w}_1-c_y \\
\\
\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2-c_y \geq \pi(x,\theta_2)\bar{w}_1+(1-\pi(x,\theta_2))\underline{w}_1-c_x \\
\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2-c_y \geq \pi(x,\theta_2)\bar{w}_2+(1-\pi(x,\theta_2))\underline{w}_2-c_x \\
\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2 \geq \pi(y,\theta_2)\bar{w}_1+(1-\pi(y,\theta_2))\underline{w}_1 \\
\\
p[\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-c_x]+ \\
\qquad (1-p)[\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2-c_y] \geq \bar{u}
\end{array}\right. \tag{P^{RN}}$$

Note that, if all the incentive compatibility constraints are ignored, the Principal can achieve, at best, a level of profit given by $p[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1))\underline{R}]+(1-p)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2))\underline{R}]-\bar{u}-pc_x-(1-p)c_y$. Thus, any contract that satisfies the IC constraints and yield this (first best level of) profit is optimal. It is immediate to verify that the compensation profile given in (7) is one such contract. ■

On the other hand, when the agent is risk averse ($a>0$) the incentive compatibility constraints cannot be ignored, agency costs are always positive, which obviously makes the case for the flexible contract worse relative to the rigid contracts.

We intend to examine whether the same property holds for also for other, less dramatic changes in the agent's degree of risk aversion, i.e., whether it is true that an increase in the level of risk aversion of the agent always leads to a lower level of profits for the principal at the optimal flexible contract, no matter what is the level of risk aversion in the initial situation.

As we could not solve explicitly for the optimal flexible contract, we have to rely on numerical solutions. We consider the following set of parameters.

$\hat{p}$	$\bar{u}$	$\bar{R}$	$\underline{R}$	c_x	c_y	$\pi(x,\theta_1)$	$\pi(x,\theta_2)$	$\pi(y,\theta_1)$	$\pi(y,\theta_2)$
.5	1	10	5	1.5	1	.8	.45	.2	.4

Figure 1 shows how the difference between the profit at the flexible contract compared to the two rigid contracts evolves as a function of risk aversion. For low levels of risk aversion, the flexible contract is preferred to the two fixed contract, while as a increases the case for the flexible contract is weaker and eventually, for a around 1.6 in this example, the fixed contract specifying the task x to the agent is preferred.

A first rough intuition for this concerns the fact that agency costs are increasing with the agent's risk aversion. To understand better the result we can look at the effects of changing the agent's risk aversion coefficient (a) on each of the three constraints above in the problem yielding the optimal flexible contract:

i. increasing risk aversion (a) makes the participation constraint, ceteris paribus, harder to satisfy ($\bar{u}$ is the certainty equivalent of $\bar{w}_1 - c_x$,$\underline{w}_1 - c_x$,$\bar{w}_2 - c_y$, $\underline{w}_2 - c_y$ and this should always increase with risk aversion.

ii. In $IC2$ the two lotteries $\bar{w}_1 - c_x$,$\underline{w}_1 - c_x$ (with probabilities $\pi(x,\theta_1), 1 - \pi(x,\theta_1)$) and $\bar{w}_2 - c_y$,$\underline{w}_2 - c_y$ (with probabilities $\pi(y,\theta_1), 1 - \pi(y,\theta_1)$) are compared. Since IC2 binds, the two lotteries must have the same utility. What is the effect of increasing risk aversion on this comparison? The first lottery exhibits more variability in its support, as well as the highest and the lowest value in the support of the values of the agent's compensation. We may expect then that a higher risk aversion decreases the utility of first lottery more than that of the second one, thus making this constraint tighter; as a consequence the variability in w_2 (needed for such constraint to hold) - and hence the distortions needed to satisfy the incentive constraints - increase.

iii. In $IC6$ the two lotteries $\bar{w}_2 - c_y$,$\underline{w}_2 - c_y$ and $\bar{w}_1 - c_y$,$\underline{w}_1 - c_y$ (with the same probabilities $\pi(y,\theta_2)$) are compared. The first lottery exhibits lower variability; when risk aversion is increased we should expect then that the utility of this lottery increases relative to that of the second one. Hence such constraint becomes weaker and the variability of w_1 at the optimal contract, needed for this constraint to hold, decreases.

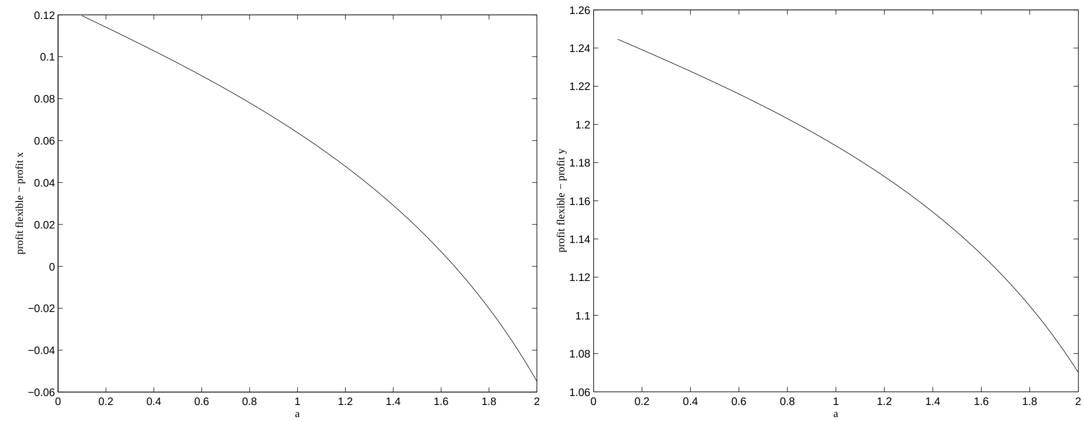

Figure 1: Profit differential between the flexible and fixed contracts as a function of risk aversion

Figure 2 illustrates the difference between the compensation with high and low output at the optimal flexible contract respectively in state θ_1 (i.e. $\bar{w}_1 - \underline{w}_1$ OK??) and θ_2 as a function of risk aversion. It shows that the spread in state θ_1 is not a monotonic function of a

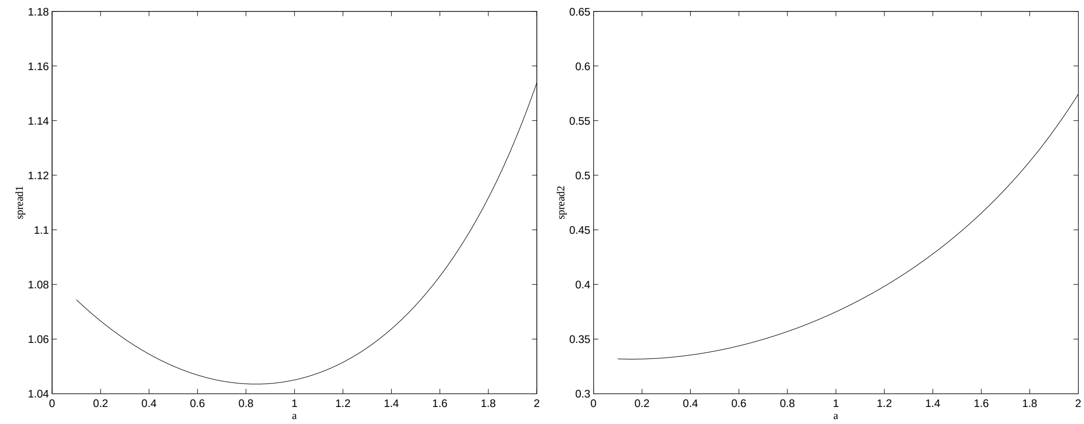

Figure 2: Wage differentials at the optimal flexible contract as a function of risk aversion

4.2.1 A shot at a formal argument on comparative statics w.r.t. a

Here want to build on the intuition given above to explain our findings in the numerical examples to provide an argument showing that when risk aversion increases the value of the flexible contract always decreases, in the sense that the wage bill increases for the Principal.

Let $\bar{z}_1 = e^{-a\bar{w}_1}$, $\underline{z}_1 = e^{-a\underline{w}_1}$, $\bar{z}_2 = e^{-a\bar{w}_2}$, $\underline{z}_2 = e^{-a\underline{w}_2}$ be the optimal solution to the program

$(\tilde{P}^T)$ with risk aversion a (call the vector z). Similarly let $\bar{z}'_1 = e^{-a'\bar{w}'_1}$, $\underline{z}'_1 = e^{-a'\underline{w}'_1}$, $\bar{z}'_2 = e^{-a'\bar{w}'_2}$, $\underline{z}'_2 = e^{-a'\underline{w}'_2}$ be the optimal solution to the program $(\tilde{P}^T)$ with risk aversion a' (call this vector z'). Define $\bar{z}^\star_1 = e^{-a\bar{w}'_1}$, $\underline{z}^\star_1 = e^{-a\underline{w}'_1}$, $\bar{z}^\star_2 = e^{-a\bar{w}'_2}$, $\underline{z}^\star_2 = e^{-a\underline{w}'_2}$ and call the vector $z^\star$.

If we can prove that $z^\star$ is feasible in program $(\tilde{P}^T)$ with risk aversion a, then, we get

$$\begin{array}{c} p(\pi(x,\theta_1)\log\bar{z}^\star_1 + (1-\pi(x,\theta_1))\log\underline{z}^\star_1) + (1-p)(\pi(y,\theta_2)\log\bar{z}^\star_2 + (1-\pi(y,\theta_2))\log\underline{z}^\star_2) \\ < \\ p(\pi(x,\theta_1)\log\bar{z}_1 + (1-\pi(x,\theta_1))\log\underline{z}_1) + (1-p)(\pi(y,\theta_2)\log\bar{z}_2 + (1-\pi(y,\theta_2))\log\underline{z}_2) \end{array}$$

This is equivalent to

$$\begin{array}{c} \log(\bar{z}_1^{\star p\pi(x,\theta_1)}\underline{z}_1^{\star p(1-\pi(x,\theta_1))}\bar{z}_2^{\star(1-p)\pi(y,\theta_2)}\underline{z}_2^{\star(1-p)(1-\pi(y,\theta_2))}) \\ < \\ \log(\bar{z}_1^{p\pi(x,\theta_1)}\underline{z}_1^{p(1-\pi(x,\theta_1))}\bar{z}_2^{(1-p)\pi(y,\theta_2)}\underline{z}_2^{(1-p)(1-\pi(y,\theta_2))}) \end{array}$$

Recall that (with obvious notation) $z = e^{-aw}$ and $z^\star = e^{-aw'}$, and replace in the expression to get (dropping the log):

$$\begin{array}{c} e^{-a(p(\bar{w}'_1\pi(x,\theta_1)+\underline{w}'_1(1-\pi(x,\theta_1)))+(1-p)(\bar{w}'_2\pi(y,\theta_2)+\underline{w}'_2(1-\pi(y,\theta_2))))} \\ < \\ e^{-a(p(\bar{w}_1\pi(x,\theta_1)+\underline{w}_1(1-\pi(x,\theta_1)))+(1-p)(\bar{w}_2\pi(y,\theta_2)+\underline{w}_2(1-\pi(y,\theta_2))))} \end{array}$$

Hence

$$\begin{array}{c} a(p(\bar{w}_1\pi(x,\theta_1) + \underline{w}_1(1-\pi(x,\theta_1))) + (1-p)(\bar{w}_2\pi(y,\theta_2) + \underline{w}_2(1-\pi(y,\theta_2)))) \\ < \\ a(p(\bar{w}'_1\pi(x,\theta_1) + \underline{w}'_1(1-\pi(x,\theta_1))) + (1-p)(\bar{w}'_2\pi(y,\theta_2) + \underline{w}'_2(1-\pi(y,\theta_2)))) \end{array}$$

Let's call Ew' and Ew the expected wage (for programme a' and programme a) as they appear in the expression above. We then get $Ew' > Ew$, which is what we want to show.

So let's proceed to try to show that $z^\star$ is feasible in program $(\tilde{P}^T)$ with risk aversion a, that is, the following system defines a non empty set:

$$\begin{cases} (IC2^\star) \;\; \pi(x,\theta_1)e^{-a(\bar{w}'_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}'_1-c_x)} \le \pi(y,\theta_1)e^{-a(\bar{w}'_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}'_2-c_y)} \\ (IC6^\star) \;\; \pi(y,\theta_2)e^{-a\bar{w}'_2} + (1-\pi(y,\theta_2))e^{-a\underline{w}'_2} \le \pi(y,\theta_2)e^{-a\bar{w}'_1} + (1-\pi(y,\theta_2))e^{-a\underline{w}'_1} \\ \\ (PC^\star) \;\; p[\pi(x,\theta_1)e^{-a(\bar{w}'_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}'_1-c_x)}] + \\ \qquad (1-p)[\pi(y,\theta_2)e^{-a(\bar{w}'_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}'_2-c_y)}] \le e^{-a\bar{u}} \end{cases}$$

knowing that

$$\begin{cases} (IC2') \;\; \pi(x,\theta_1)e^{-a'(\bar{w}'_1-c_x)} + (1-\pi(x,\theta_1))e^{-a'(\underline{w}'_1-c_x)} = \pi(y,\theta_1)e^{-a'(\bar{w}'_2-c_y)} + (1-\pi(y,\theta_1))e^{-a'(\underline{w}'_2-c_y)} \\ (IC6') \;\; \pi(y,\theta_2)e^{-a'\bar{w}'_2} + (1-\pi(y,\theta_2))e^{-a'\underline{w}'_2} = \pi(y,\theta_2)e^{-a'\bar{w}'_1} + (1-\pi(y,\theta_2))e^{-a'\underline{w}'_1} \\ \\ (PC') \;\; p[\pi(x,\theta_1)e^{-a'(\bar{w}'_1-c_x)} + (1-\pi(x,\theta_1))e^{-a'(\underline{w}'_1-c_x)}] + \\ \qquad (1-p)[\pi(y,\theta_2)e^{-a'(\bar{w}'_2-c_y)} + (1-\pi(y,\theta_2))e^{-a'(\underline{w}'_2-c_y)}] = e^{-a'\bar{u}} \end{cases}$$

Consider (IC6') and write it:

$$\pi(y,\theta_2)u(\bar{w}_2') + (1-\pi(y,\theta_2))u(\underline{w}_2') = \pi(y,\theta_2)u(\bar{w}_1') + (1-\pi(y,\theta_2))u(\underline{w}_1')$$

where $u(x) = -e^{-a'x}$. This says that the lottery giving $u(\bar{w}_2')$ with probability $\pi(y,\theta_2)$ and $u(\underline{w}_2')$ with probability $1-\pi(y,\theta_2)$ has the same mean that the lottery giving $u(\bar{w}_1')$ with probability $\pi(y,\theta_2)$ and $u(\underline{w}_1')$ with probability $1-\pi(y,\theta_2)$. Furthermore the first lottery second order stochastically dominates the second one [i.e., the second is a mean preserving spread of the first] given that $u(\bar{w}_1') \geq u(\bar{w}_2') > u(\underline{w}_2') \geq u(\underline{w}_1')$.

Consider now the utility function $f(x) = -(-x)^{a/a'}$. f is a convex function since $a/a' < 1$. Hence a decision maker with a convex utility function who has to compare two lotteries, one of them being a mean preserving spread of the other, will favor the former. Hence,

$$\pi(y,\theta_2)f\circ u(\bar{w}_2') + (1-\pi(y,\theta_2))f\circ u(\underline{w}_2') < \pi(y,\theta_2)f\circ u(\bar{w}_1') + (1-\pi(y,\theta_2))f\circ u(\underline{w}_1')$$

Since $f\circ u(x) = -(e^{-a'x})^{a/a'} = -e^{-ax}$, this yields:

$$\pi(y,\theta_2)e^{-a\bar{w}_2'} + (1-\pi(y,\theta_2))e^{-a\underline{w}_2'} > \pi(y,\theta_2)e^{-a\bar{w}_1'} + (1-\pi(y,\theta_2))e^{-a\underline{w}_1'}$$

which shows that $(IC6^\star)$ is violated.

The opposite argument holds for (PC*) which is then satisfied when (PC) is satisfied: the l.h.s. is a lottery that has the same mean as the (degenerate) lottery giving $\bar{u}$ for sure; this last degenerate lottery second order stochastically dominates the first lottery and hence the same reasoning can be made: increasing risk aversion implies that the expected utility of the lottery is higher than the expected utility of the degenerate lottery.

5 The Choice between flexible and rigid contracts in the presence of ambiguity

We examine now the case where, at the time the contract is written the information available to the parties concerning the likelihood of the events θ_1, θ_2 is not precise enough for them to have a sharp probability belief over them. This appears rather natural if we are to think of such events as possible contingencies not clearly defined at the contracting stage.

We use here the analysis of Gajdos, Tallon, Vergnaud (2005) (GTV) to model this situation: we assume therefore there is a given set of possible values for p given by the interval $[\underline{p}, \bar{p}]$ in which the "true parameter" p lies. Let $\hat{p} \equiv \frac{\underline{p}+\bar{p}}{2}$. In GTV, a general decision criterion is axiomatized. This specializes in the present case to the following criterion: a decision maker evaluates an act by taking a linear combination between its minimal expected utility according to the information available to the agent (as described by the set of possible beliefs entertained by the agents) and the expected utility with respect to $\hat{p}$. In the simple case we consider here (only two states), this criterion amounts to taking the minimal expected utility with respect to a p in $[\underline{p}+\psi, \bar{p}-\psi]$, where ψ is a measure of uncertainty aversion. Call ψ the uncertainty aversion of the Principal and α the uncertainty aversion of the Agent. The Multiple prior model of Gilboa and Schmeidler (..), though not directly comparable given its subjective nature (which makes it hard to interpret the fact that Principal and Agent would have similar or even related "beliefs") would be that $\alpha = \psi = 0$ and would be interpreted as extreme ambiguity aversion from the two parties. The other extreme case is $\alpha = \psi = \hat{p}$ which corresponds to expected utility (ambiguity neutrality) from the two parties.

We will analyze the consequences of the degree of uncertainty aversion of principal and agent for the form of the optimal flexible contract and the choice between flexible and rigid contracts, comparing them with the consequences of the agent's degree of risk aversion we found in the previous section. In this regard, it is important to point out that while the degree of risk aversion concerns both the uncertainty concerning the environment where the decisions will be made (described by θ) as well as the uncertainty over the output realization, the degree of uncertainty aversion only concerns the first source of uncertainty (θ).

5.1 Risk neutral but uncertainty averse agent and principal

The optimal flexible contract when in this case is obtained as solution of the following programme:

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad \min_{p\in[\underline{p}+\psi,\bar{p}-\psi]} \{p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] + (1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]\}$$

$$\left\{\begin{array}{l}
\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1 \geq \pi(x,\theta_1)\bar{w}_2+(1-\pi(x,\theta_1))\underline{w}_2 \\
\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1 - c_x \geq \pi(y,\theta_1)\bar{w}_2+(1-\pi(y,\theta_1))\underline{w}_2 - c_y \\
\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1 - c_x \geq \pi(y,\theta_1)\bar{w}_1+(1-\pi(y,\theta_1))\underline{w}_1 - c_y \\
\\
\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2 - c_y \geq \pi(x,\theta_2)\bar{w}_1+(1-\pi(x,\theta_2))\underline{w}_1 - c_x \\
\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2 - c_y \geq \pi(x,\theta_2)\bar{w}_2+(1-\pi(x,\theta_2))\underline{w}_2 - c_x \\
\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2 \geq \pi(y,\theta_2)\bar{w}_1+(1-\pi(y,\theta_2))\underline{w}_1 \\
\\
\min_{p\in[\underline{p}+\alpha,\bar{p}-\alpha]} \{p[\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1 - c_x] + \\
\qquad (1-p)[\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2 - c_y]\} \geq \bar{u}
\end{array}\right. \qquad (P^{RN,UA})$$

We can show that, as in the case where both principal and agent are uncertainty neutral, the first best optimal contract (obtained as solution of the problem of maximizing the principal's expected profits as above but subject only to the participation constraint) is an admissible solution also of problem ($P^{RN,UA}$), i.e. when the IC constraints are also taken into account:

Consider the compensation profile in (7): as shown in Claim 2 this constitutes the optimal flexible contract when the agent is risk neutral and both agent and principal have a single, common prior (equivalently, are ambiguity neutral). Note such contract is also feasible here, when principal and agent are ambiguity averse: the incentive constraints do not depend on p, they are thus unaffected by the degree of uncertainty aversion, and, since the agent receives the same utility in θ_1 and θ_2, the participation constraint is also satisfied (whatever is the value of p). The principal's expected profits at this contract are given by

$$\min_{p\in[\underline{p}+\psi,\bar{p}-\psi]} \left\{p[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1))\underline{R}-\bar{u}-c_x]+(1-p)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2))\underline{R}-\bar{u}-c_y]\right\},$$

and the above expression is evaluated at $\underline{p}+\psi$ if $(\pi(x,\theta_1)-\pi(y,\theta_2))\left[\bar{R}-\underline{R}\right]>\Delta c$, at $\bar{p}-\psi$ if the strict inequality holds in the other direction, and at any p in the interval if the two terms are equal.

Claim 3 *When the agent is risk neutral but principal and agent are both uncertainty averse, the optimal flexible contract is still first best optimal.*
Furthermore, if the agent is at least as ambiguity averse as the principal, the optimal compensation is the same as the one derived in (7), under uncertainty neutrality.
The presence of uncertainty aversion expands the range of parameter values for which the flexible contracts dominates the fixed one.

Proof. If the agent is at least as risk averse as the principal, at the compensation profile given by (7) the beliefs of the principal and the agent (since the latter receives the same utility in θ_1 and θ_2) are aligned; hence we can say the contract implements the first best level of profits when both principal and agent are risk neutral and with a single belief, equal to $\underline{p}+\psi$ if the parameter values are such that if $(\pi(x,\theta_1)-\pi(y,\theta_2))\left[\bar{R}-\underline{R}\right]>\Delta c$, to $\bar{p}-\psi$ otherwise. It is easy to verify that any other specification of the agent's compensation inducing the use of the these beliefs cannot yield a higher level of the profits. At the same time, specifications inducing different beliefs for principal and agent yield a lower level of profits (because the agent will be paid more in the state he values less, relatively to the principal, and doing this reduces the profits in the state the principal values more).

If the principal is more ambiguity averse than the agent, then $[\underline{p}+\alpha,\bar{p}-\alpha]\subset[\underline{p}+\psi,\bar{p}-\psi]$. In such case the compensation profile given by (7) remains feasible and induces the same level of expected profits as above (again evaluated at $\underline{p}+\psi$ - resp. $\bar{p}-\psi$). In this situation however, a different compensation profile, insuring the principal rather than the agent, if feasible, would induce a higher level of profits (the same expression but now evaluated at $\underline{p}+\alpha$ - resp. $\bar{p}-\alpha$). This requires

$$\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-(\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2)\geq(\pi(x,\theta_1)-\pi(y,\theta_2))\left[\bar{R}-\underline{R}\right]$$

It remains then to show that this condition is consistent with the incentive compatibility constraints being satisfied.

Profits at a rigid contract are on the other hand always evaluated at $\underline{p}+\psi$ for the x contract (since $(\pi(x,\theta_1)-\pi(x,\theta_2))\left[\bar{R}-\underline{R}\right]>0$) and at $\bar{p}-\psi$ for the y contract (since $\pi(y,\theta_1)-\pi(y,\theta_2))\left[\bar{R}-\underline{R}\right]<0$). Since the latter are always lower than if they were evaluated at $\underline{p}+\psi$ this establishes the result. ■

5.2 Risk averse and uncertainty averse agent

The maximization problem the principal has to solve when the agent is risk averse (still assuming the CARA form for agent's utility function) is

$$\begin{aligned}\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad \min_{p\in[\underline{p}+\psi,\bar{p}-\psi]} &\left\{p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)]\right.\\ &\left.+(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]\right\}\end{aligned}$$

$$\begin{cases} (IC1) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}\leq\pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)}\\ (IC2) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}\leq\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}\\ (IC3) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}\leq\pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}\\ (IC4) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}\leq\pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)}\\ (IC5) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}\leq\pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)}\\ (IC6) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}\leq\pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}\\ \\ (PC) & \min_{p\in[\underline{p}+\alpha,\bar{p}-\alpha]}\left\{p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}]+\right.\\ & \left.\quad(1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}]\right\}\leq e^{-a\bar{u}} \end{cases} \quad (P^{UA})$$

Note that the degree of uncertainty aversion of the agent only matters for the specification of the participation constraint, not of the incentive compatibility constraints, which are independent of p (and in fact the same as the ones of its ambiguity neutral equivalent (see section 3)). We can show in fact that the characterization of the optimal flexible contract is analogous to the one derived for the case where there was no uncertainty aversion:

Claim 4 *At the optimal flexible contract when the agent is risk averse and agent and principal are uncertainty averse, provided the principal is less ambiguity averse than the agent, the only IC constraints that are binding are (IC2) and (IC6) and wages are such that* $\bar{w}_1\geq\bar{w}_2\geq\underline{w}_2\geq\underline{w}_1$ *(with* $\bar{w}_1>\underline{w}_1$*). The optimal contract is then the solution to:*

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad \min_{p\in[\underline{p}+\psi,\bar{p}-\psi]} p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]$$

$$\begin{cases} (IC2) \;\; \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} = \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \\ (IC6) \;\; \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} = \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)} \\ \\ (PC) \;\; \min_{p\in[\underline{p}+\alpha,\bar{p}-\alpha]} \{p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}]+ \\ \qquad (1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}]\} \le e^{-a\bar{u}} \\ \\ (W) \;\; \bar{w}_1 \ge \bar{w}_2 \end{cases} \tag{$P^{red,UA}$}$$

Proof. Careful inspection of steps 1 to 9 of the proof of Proposition 1, considering the case without ambiguity, shows that the same argument goes through with ambiguity. Note in fact that such steps rely either on a manipulation of the IC constraints, which are unchanged in problem ($P^{red,UA}$), or on some optimality arguments. For the latter case, two types of arguments can be distinguished. The first one concerns changes of the compensation within a given state; this does not depend on the beliefs over θ and only relies on the concavity of the agent's utility function. On the other hand, the second type of argument considers changes of wages - and hence utility - across states and clearly depends on the beliefs over θ. This only appears in establishing case 1 of step 3. We show here that the same line of reasoning holds with ambiguity aversion, provided the principal is less ambiguity averse than the agent.

Let's establish that (IC2) has to be binding in the case in which w_2 is constant. Assume (IC2) is slack and w_2 constant. This implies that the utility of the agent is higher in state θ_1 than in state θ_2. He thus uses $\underline{p}+\alpha$ in his computation of expected utility. Consider again (an infinitesimal change) $d\bar{w}_1 < 0$, $d\underline{w}_1 = 0$ and $dw_2 > 0$ but now defined using the belief $\underline{p}+\alpha$: $d\bar{w}_1 = -\frac{(1-\underline{p}-\alpha)}{(\underline{p}+\alpha)}\frac{e^{-a(w_2-c_y)}}{e^{-a(\bar{w}_1-c_x)}}dw_2$, so that the participation constraint still holds. We will show that this change increases the principal's profit. Let $p \in [\underline{p}+\psi, \bar{p}-\psi]$ be the probability used by the principal in his evaluation of the expected profit. The change in profit is then equal to

$$\left[p\frac{(1-\underline{p}-\alpha)}{(\underline{p}+\alpha)}\frac{e^{-a(w_2-c_y)}}{e^{-a(\bar{w}_1-c_x)}}-(1-p)\right]dw_2$$

Given that $dw_2 > 0$ this term is positive whenever $\frac{e^{-a(w_2-c_y)}}{e^{-a(\bar{w}_1-c_x)}} > \frac{(\underline{p}+\alpha)}{(1-\underline{p}-\alpha)}\frac{(1-p)}{p}$. The term on the right hand side is smaller than 1 whenever $p > \underline{p}+\alpha$, always true when $\psi \ge \alpha$, i.e., under the assumption that the principal is less ambiguity averse than the agent (or they have the same aversion). In that case the argument given in the case without ambiguity, showing that $\frac{e^{-a(w_2-c_y)}}{e^{-a(\bar{w}_1-c_x)}} > 1$, is enough to show that the considered change increases the principal's profit here as well. ■

Since $\bar{w}_2 \ge \underline{w}_2$, the fact that (IC2) and (IC6) hold with equality, together with the fact that $\pi(y,\theta_2) > \pi(y,\theta_1)$ imply, as we saw in Section 3, that the agent's utility in state θ_2 is higher than or equal (equality holds if and only if $\bar{w}_2 = \underline{w}_2$) to his utility in state θ_1. Hence the Agent will use beliefs $\bar{p} - \alpha$ (if his utility is strictly higher in state θ_2) or any p in $[\underline{p} + \alpha, \bar{p} - \alpha]$ (in case the utilities are equal, in which case the value of p is irrelevant) in his evaluation of minimal expected utility.

We show next that,

Claim 5 *If the level of uncertainty aversion is sufficiently higher for the agent than for the principal, at the optimal contract the principal fully insures the agent across the θ states, i.e. $\bar{w}_2 = \underline{w}_2$ and the agent receives the same utility in θ_1 and in θ_2.*

Proof. We consider for simplicity the case where the principal is ambiguity neutral: $\bar{p} - \psi = \underline{p} + \psi = \hat{p}$.

A possible solution of problem $(P^{red,UA})$ is given, as we argued, by $\bar{w}_2 = \underline{w}_2$. In that case, (IC2) implies that the agent has the same level of utility in states θ_1 and θ_2. Thus there is no loss of generality in choosing the agent's beliefs to be the same as the principal's and equal to $\hat{p}$. It is then easy to show, by a similar argument as in Section 3, that the optimal contract solving problem $(P^{red,UA})$ such that $\bar{w}_2 = \underline{w}_2$ is given by $\bar{w}_2 = \underline{w}_2 = \bar{u} + c_y$ and $\bar{w}_1, \underline{w}_1$ such that:

$$\begin{aligned} \pi(x,\theta_1)e^{-a(\bar{w}_1 - c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1 - c_x)} &= e^{-a\bar{u}} \\ \pi(y,\theta_2)e^{-a(\bar{w}_1 - c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_1 - c_y)} &= e^{-a\bar{u}} \end{aligned}$$

An alternative possibility is a contract such that $\bar{w}_2 > \underline{w}_2$. In this case, as we saw in in Section 3, the IC constraints imply that the principal will not fully insure the agent, whose utility in state θ_1 is strictly smaller than in state θ_2. The agent therefore uses beliefs $p = \bar{p} - \alpha$ so that there is a difference between the beliefs which are "effectively" used by the two parties.The optimal contract is given by the solution to the programme:

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad \widehat{p}[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ +(1-\hat{p})[\pi(y,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]$$

$$\begin{cases} (IC2) \ \ \& \ \ (IC6) \text{ with equality} \\ (PC) \ \ (\bar{p}-\alpha)[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}] + \\ \qquad (1-\bar{p}+\alpha)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}] \leq e^{-a\bar{u}} \\ (W) \ \ \bar{w}_1 \geq \bar{w}_2 \end{cases}$$

and is then analogous to the optimal compensation we found in Proposition 3 when principal and agent have single, identical prior beliefs over θ.

■

We also illustrate the above results in the set-up of some numerical simulations (for the case of an ambiguity neutral principal). Assume the following values for our set of parameters:

a	$\hat{p}$	$\bar{u}$	$\bar{R}$	$\underline{R}$	c_x	c_y	$\pi(x,\theta_1)$	$\pi(x,\theta_2)$	$\pi(y,\theta_1)$	$\pi(y,\theta_2)$
1	.5	1	10	5	1.5	1	.8	.45	.2	.4

The agent's ambiguity aversion is measured by α. At the candidate optimal contract such that $\bar{w}_2 = \underline{w}_2$ the agent's beliefs are irrelevant and can be set with no loss of generality equal to those of the principal, given by $\hat{p}$. On the other hand, at the candidate optimal contract such that $\bar{w}_2 > \underline{w}_2$ the agent uses beliefs $\bar{p} - \alpha$ (note that $\bar{p} - \alpha > \hat{p}$). The lower is α, the higher is the ambiguity aversion and hence the difference $\bar{p} - \alpha - \hat{p}$ between the beliefs of principal and agent in the latter contract.

In the diagram below we illustrate how the optimal flexible contract varies with $\varepsilon \equiv \bar{p} - \alpha - \hat{p}$, i.e. with the ambiguity aversion of the agent (as explained above, the higher ε the higher the ambiguity aversion), for $\bar{p} = .95$. The way to read figure 3 is as follows: as ε goes from 0 (ambiguity neutrality) to its maximal level, given by $\bar{p} - \hat{p} = .45$, the difference, at the optimal contract, between the utility of the agent in state θ_1 and that in state θ_2 decreases and eventually (at around $\varepsilon = .35$), these two quantities are the same. Figure 4 represents the spread in wages (i.e. $\bar{w}_i - \underline{w}_i$) in state θ_1 and θ_2 respectively.

As already argued in the previous section, rigid contracts are unaffected by the presence of uncertainty aversion and profits are given by the value derived in Section 4 when the principal's beliefs are given by $\hat{p}$. As a consequence, we conjecture that the effect of a higher degree of uncertainty aversion of the agent relative to the principal on the choice between flexible and rigid contracts is somewhat opposite to the effect of the degree of risk aversion.

6 References

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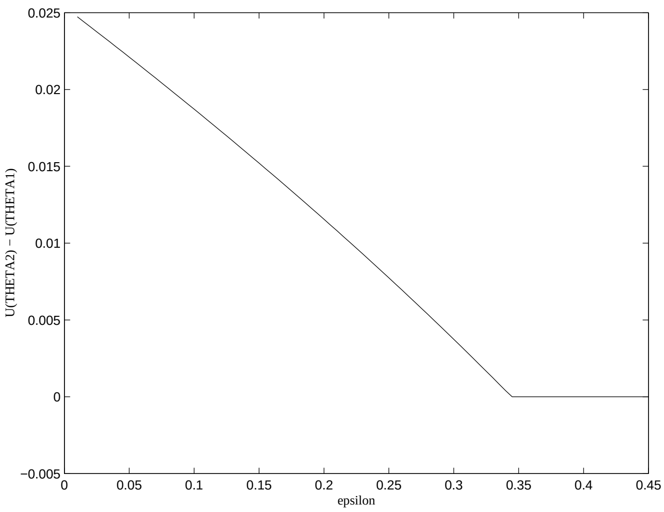
Figure 3: Utility differential at the optimal contract as a function of ambiguity aversion



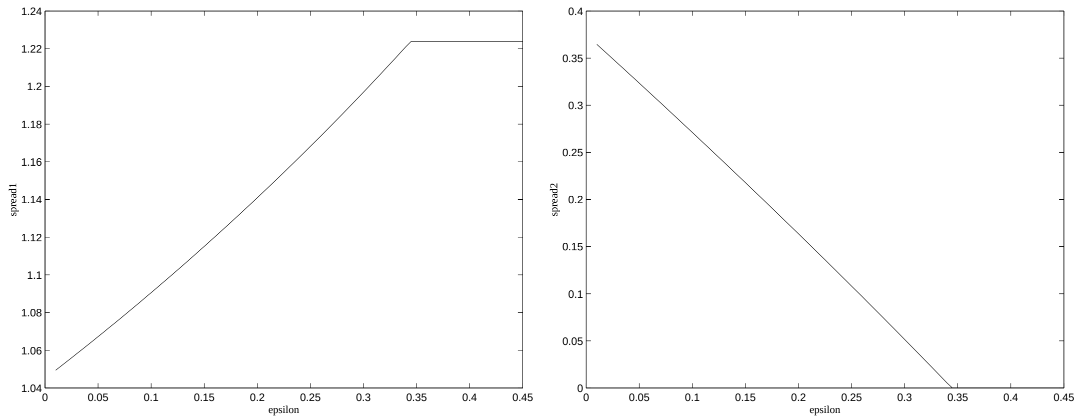


Figure 4: Wage differentials at the optimal contract as a function of ambiguity aversion